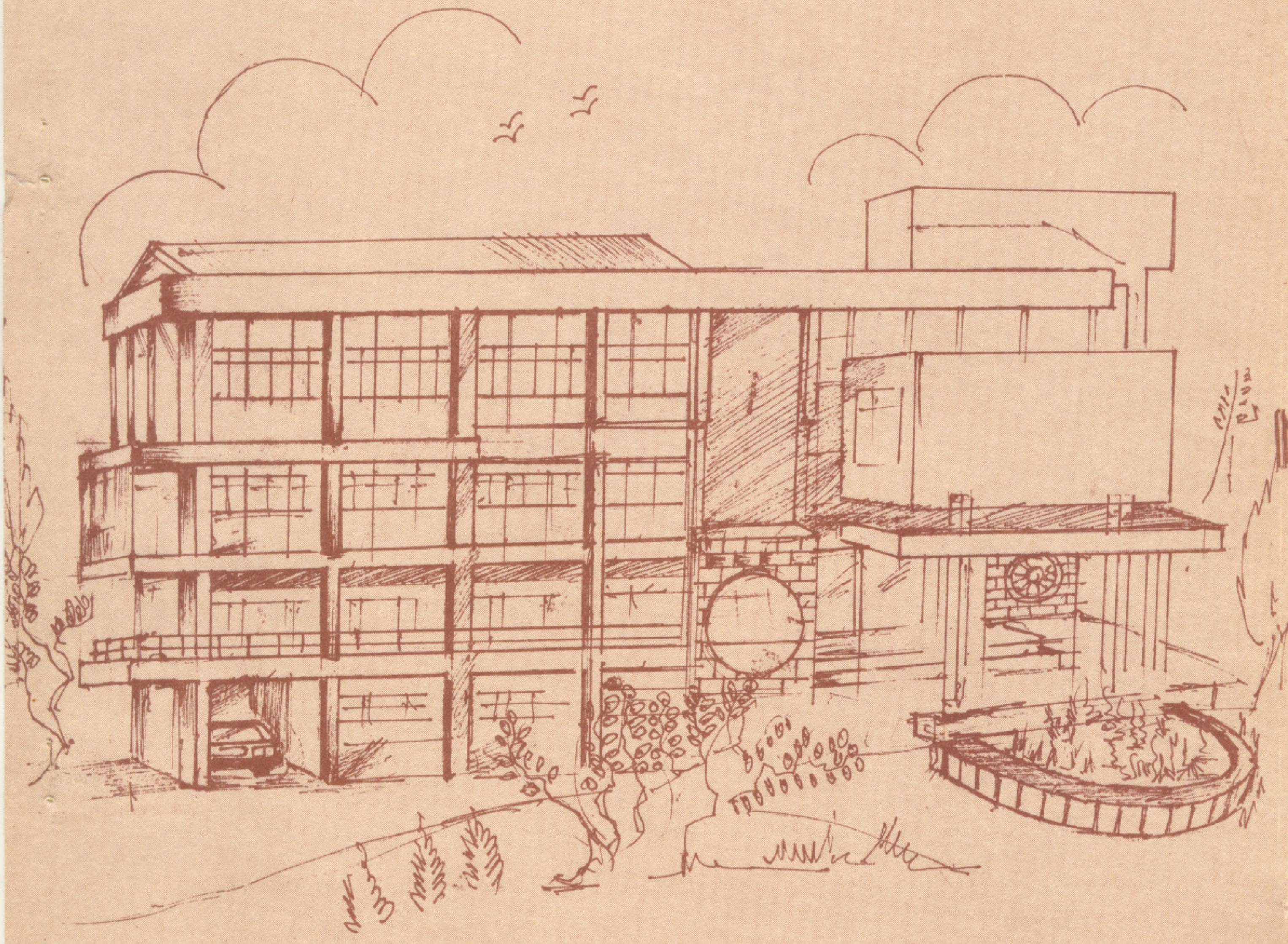
T.A. PAI MANAGEMENT INSTITUTE
MANIPAL

TAPMI

Working Paper Series

No. ~~21~~ 20



Radial & Non-radial Measures of Technical Efficiency in Data Envelopment Analysis: An Empirical Comparison

By

Dr Biresh K Sahoo
Assistant Professor

TAPMI WORKING PAPER SERIES NO. 2000/03

T. A. PAI MANAGEMENT INSTITUTE
MANIPAL - 576 119

Radial and Non-radial Measures of Technical Efficiency in Data Envelopment Analysis: An Empirical Comparison

Abstract: *In this paper we attempt to provide theoretical justifications for distinguishing and comparing the two definitions of technical efficiency offered by Farrell and Russell, and empirically examine the two efficiencies in the context of Indian Pig Iron and Sponge Iron Industry. The empirical analysis shows that the technical efficiency ratings of the problematic DMUs (those operating not in the efficient subset of the production technology) in the radial measure of Farrell have drastically gone down in the non-radial Russell measure. The weighted Russell measure is proved to be useful when slack exists and when different inputs impact production differently. An effort has also been made to see how far the extremely efficient DMUs are robust in the face of input perturbations. The results indicate that only two units are robust in the face of input increase, under the assumption of constant returns to scale.*

Key Words: *DEA; Efficiency; Peer Group; Excess Slacks*

1. Introduction

Technical efficiency is basically a success indicator and performance measure by which decision making units (DMUs) are evaluated in a competitive set up. Measuring efficiency accurately and separating its effects from the effects of production environments enable one to explore the sources of efficiency differentials. Knowledge of such sources is highly essential to both the public and private institutions especially for their policies that are designed to improve performance (Lovell, 1993).

The term input/output *technical efficiency* is defined in the literature in two alternative ways: in terms of either radial orientation or non-radial orientation. The neo-classical idea in terms of input radial orientation, known widely as the Farrell measure of efficiency, is that technical efficiency is one minus maximum feasible equiproportionate reduction in all inputs that allows the continued production of output (Debreu, 1951, Farrell, 1957). Farrell's method measures technical efficiency relative to an *isoquant* of an input set rather than to its efficient subset, a practice that can lead to identifying a DMU as being technically efficient when it is not. Further, the assumption of equiproportionate reduction in all inputs has been taken as a rule though there is no *a priori* reason to measure technical efficiency radially even for homothetic technologies. This means that there are no trade off between inputs, which is counter intuitive. Price information would generally indicate that the opportunity costs of consumption of one input over another are not the same (Morey *et al.*, 1990). Consequently it might be optimal to reduce inputs in non-equal proportions reflecting their differing opportunity costs. A non-radial measure of technical efficiency implies that a DMU is efficient if an increase in an output requires reduction in at least one other output or an increase in at least one input, and if a reduction in any input requires an increase in at least one other input or a reduction in at least one other output (Koopmans, 1951). Thus a technically inefficient DMU could produce the same output with less of at least one input, or could use the same inputs to produce more of at least one output. This measure relates any DMU to a DMU of its efficient subset, which is clearly a subset of *isoquant* for each output.

The two definitions of technical efficiency are same only when the underlying production technology that comprises only properly enveloped units is efficient for each output. However, the production technology can also be a more general concept to include DMUs of two types: properly enveloped and not properly enveloped ones. It is thus weak efficient and is characterized by *isoquant* for each output. If this is the case, then the two definitions are likely to be different and hence they cannot be used interchangeably. However, we have not come across any papers in the literature that empirically distinguish the above two definitions of technical efficiency. The failure to distinguish between the two could lead to erroneous inferences concerning technical efficiency. It has been argued (Färe and Lovell, 1978) that the non-radial measure of efficiency is well behaved under the less restrictive technological assumption and is justified by the dependence on Farrell's ideas of a grewing number of empirical studies dealing with estimation of production frontiers, which may not satisfy all the strong technological assumptions under which Farrell's ideas are valid. This paper aims at bringing out the distinction between these two concepts of technical efficiency, based on an empirical study.

Currently, there are two sets of methods for estimating efficiency: econometric frontier production function, and data envelopment analysis (DEA). The econometric approach to frontier estimation fails to distinguish between the two concepts of technical efficiency though there have been significant improvements in its methodology during the past thirty years. Rather, the literature give an illusion that the two concepts are one and the same. The non-parametric approach of DEA, originally pioneered by Charnes *et al.* (1978, 1979), is known as the CCR model after Charnes, Cooper, and Rhodes. It is extended later by Banker *et al.* (1984) and this extended model is known as the BCC model after Banker, Charnes, and Cooper. These non-parametric models have managerial decision-making orientation and are geared to making the best use of on-hand rescurces within the public or private sector organizations. These models also help in differentiating and comparing between the two above-noted definitions of technical efficiency. Since DEA is very sensitive to measurement error or noise, especially where these factors affect what turn out to be DEA efficient units, we are interested in changes of inputs and outputs of efficient DMUs preserving efficiency.

The structure of this paper is as follows: Section 2 gives an overall theoretical background of production technology and describes how DEA can be utilized to (1) measure and compare technical efficiency in both Farrell and Koopmans sense, and (2) study sensitivity analysis of extremely efficient DMUs. Section 3 deals with the empirical application to real-life data from Indian Pig Iron and Sponge Iron industry for the year 1995. Section 4 gives a summary of the work.

2. The Theoretical Background

2.1 The Reference Technology

Let us consider a sample of J DMUs from an industry producing a vector of N outputs (y) from a vector of

M inputs (x). Let B denote the $(J \times N)$ matrix of observed outputs and let A denote the $(J \times M)$ matrix of observed inputs. The individual elements of N, denoted y^j_n measure the quantity of the n^{th} output produced by the j^{th} DMU, while individual elements of M, denoted x^j_m, measure the employment level of the m^{th} input used by the j^{th} DMU. A production technology that transforms the input vector x to the output vector y can be represented by the output set $P(x)$, the input set $L(y)$, or the graph of the technology $GR(x,y)$.

The output set, $P(x)$, denotes the set of all output vectors that are obtainable from the input vector x. Symbolically,

$$P(x) = \{y : (x, y) \in GR\} \quad \text{....(1)}$$

The input set, $L(y)$, denotes the collection of all input vectors x that yield at least the output vector y. Mathematically,

$$L(y) = \{x : (x, y) \in GR\} \quad \text{.....(2)}$$

The graph of technology, $GR(x, y)$, is the collection of all feasible input-output vectors, i.e.,

$$\begin{aligned} GR(x, y) &= \{(x, y) : x \in L(y)\} \\ &= \{(x, y) : y \in P(x)\} \end{aligned} \quad \text{.....(3)}$$

Let us further assume that $L(y)$ has the following properties:

1. $L(y)$ exhibits strong disposability (also called free disposability) with respect to the inputs. That is, if $x \in L(y)$ and $y^* \leq y$, then $x \in L(y^*)$. ($y^* \leq y$ represents that every element of the vector y^* is smaller than or equal to the corresponding element of the vector y).

2. $L(y)$ exhibits strong disposability with respect to the outputs. That is, if $x \in L(y)$ and $x^* \leq x$, then $x^* \in L(y)$.

3. $L(y)$ is a convex set. That is, $x^0, x^1 \in L(y) \Rightarrow \gamma x^0 + (1-\gamma) x^1 \in L(y)$, $(0 \leq \gamma \leq 1)$.

These assumptions also hold good for the graph of technology and the output set.

There are two subsets of $L(y)$ which are of special interest. First, the *isoquant* for output bundle y can be defined as

$$\overline{L}(y) = \{x : x \in L(y); \theta x \notin L(y) : \theta \in [0,1)\} \quad \text{......(4)}$$

The *isoquant* consists of all input bundles such that any proportionate reduction in all the inputs would render the specified output infeasible. But a reduction in one or several inputs, but not all, is not ruled out.

The other subset is the `efficient' bundles of outputs which is itself a subset of the *isoquant*. This can be defined as

$$L^{*}(y)=\{x: x\in L(y); x'\leq x, x'\notin L(y)\} \quad ......(5)$$

It implies that when an input-output vector is efficient, it is the minimal input vector, in the sense that any reduction in any individual input will render the specified output infeasible.

All the efficiency measures have, in common, the feature that they gauge the efficiency of an individual DMU relative to a `best practice' frontier. Such a frontier is constructed as a piecewise-linear envelopment of the data generated by the set of all DMUs in the referent group. So if the input set *L(y)* is assumed to be piecewise linear, then the technology satisfying constant returns to scale (*CRS*) and assumptions made earlier with it can be written as

$$L(y:CRS,S)=\{x: y\leq \lambda B, \lambda A\leq x\} \quad ......(6)$$

and similarly, the technology with similar set of assumptions satisfying variable returns to scale (VRS) can be written as

$$L(y:VRS,S)=\{x: y\leq \lambda B, \lambda A\leq x, \lambda I=1\} \quad ......(7)$$

where *CRS* and *VRS* denote constant returns to scale and variable returns to scale respectively, *S* denotes strong disposability and *I* is the sum (column) vector with 1 as the value of each of its elements. A similar piecewise-linear frontier can be constructed for the output set and the graph of the technology.

2.2 Farrell Efficiency

The function

$$F_j(y^{j^*},x^{j^*})=\min\{\theta: \theta x\in L(y^j:CRS,S)\}$$

is called the *(CRS, S)* Input Measure of Technical and Scale Efficiency of DMU$_{j^*}$ $(j^*\in J)$. θ can be computed as the solution for the following linear programming problem (Charnes *et al.*, 1978):

$$F_i(y^{j^*},x^{j^*}:CRS,S)=\min_{\lambda,\theta}\theta \quad .....(8)$$

$$s.t.\, y^{j^*}\leq \lambda B$$

$$\lambda A\leq \theta x^{j^*}$$

$$\lambda\geq 0$$

This describes the Farrell measure of technical and scale efficiency. It strives to maximize the proportional decrease in all inputs while remaining within the envelopment surface (production technology set). The *(CRS,S)* measure $F_i(y^{j^*}, x^{j^*}: CRS,S)$, illustrated in Figure 1, measures the efficiency of x^{j^*} in the production of y^{j^*} by computing the ratio of the largest possible contraction of x^{j^*} in $L(y^{j^*} : CRS,S)$ to itself i.e., $F_i(y^{j^*}, x^{j^*}) = F_i(y^{j^*}, x^{j^*} : CRS,S)$. x^{j^*} x^{j^*}. This clearly shows $F_i(y^{j^*}, x^{j^*})$ to be the ratio of two distances along the same ray. Clearly, a proportional decrease is possible until at least one of the excess input variables is reduced to zero. So a score of unity for θ indicates technical efficiency in the Farrell sense even if there are slacks in some of the inputs (but not all) in the input vector x^{j^*}. The point R $(\theta^* x^{j^*}_1, \theta^* x^{j^*}_2)$ in Figure 1 has Farrell efficiency rating of 1 though there remains slack in the input x_1.

INSERT Figure 1

As pointed out by Färe and Lovell (1978) there are some difficulties with the Farrell's measure. Firstly, it measures technical efficiency relative to an *isoquant* rather than to an efficient subset - a practice that can lead to the identification of a unit as being technically efficient when it is not. Secondly, it is constant which rules out factor substitutions between inputs. This assumption is always maintained even for homothetic technologies. In spite of these limitations, Farrell measure of technical efficiency has been widely used in the empirical studies since it satisfies several nice properties such as first, it is homogeneous of degree -1 in inputs, second, it is weakly monotonically decreasing, and finally it is invariant with respect to changes in units of measurement, which are noted originally by Shephard (1970) and in an efficiency measurement context, by Färe and Lovell (1978), Bol (1986, 1988) and Russell (1985, 1988, 1990).

2.3 Koopmans Efficiency

The problem arises in the CCR model regarding the efficiency score when DMUs are not properly enveloped, i.e., when some DMUs are operating just on and/or above the horizontal line of the *isoquant* in Figure 1. Let us take the input vector R $(\theta^* x_1^{j^*}, \theta^* x_2^{j^*})$ in Figure 1. It is efficient in the CCR model even though there is slacks in input x_1. Various models (Charnes *et al.* (1992), Chang and Guh (1991), Färe and Lovell (1978), Green *et al.* (1996) and Charnes *et al.* (1987)) are suggested for measuring the efficiency of problematic DMUs operating outside the efficient subset cone generated by the points C and D in the technology set. We have used the Russell measure (Färe and Lovell (1978), Russell (1985), Färe *et al.* (1994)) which is basically non-radial in nature. It seeks, for each DMU, the maximum feasible contraction in all inputs for a given output and maximum feasible expansion in all outputs with given inputs. This model labels a DMU as being efficient if and only if it belongs to the efficient subset $L^*(y)$ of input set $L(y)$, i.e., if there is no input/output slack.

The function

$$RM_i(y^{j^*}, x^{j^*} : CRS,S) = \min \sum_{m=1}^{M} \theta_m / M : (\theta_1 x_1^{j^*}, \theta_2 x_2^{j^*}, \ldots\ldots, \theta_m x_m^{j^*}) \in L(y^j : CRS,S)$$

is called the *(CRS, S)* Russell Measure of Technical and Scale Efficiency of DMU$_{j^*}$. This measure can be computed as the solution to the linear programming problem (Färe *et al.*, 1994):

$$RM_i(y^{j^*}, x^{j^*} : CRS, S) = (1/M)\min_{\lambda,\theta} \sum_{m=1}^{M} \theta_m \qquad \ldots\ldots(9)$$

$$s.t.\ y_n^{j^*} \leq \sum_{j=1}^{J} \lambda^{j^*}{}_j y^j{}_n, n = 1,2,\ldots\ldots,N$$

$$\sum_{j=1}^{J} \lambda^{j^*}{}_j x^j{}_m \leq \theta_m x^{j^*}_m, m = 1,2,\ldots\ldots,M$$

$$0 < \theta_m \leq 1, \lambda_j \geq 0$$

This measure is illustrated in Figure 1. As shown in the figure, the Russell measure allows for a non-proportional reduction in each positive input and permits an input vector to shrink all the way back to the efficient subset. Let us consider the input vector $P\ (x^{j^*})$ in the technology set $L(y^j : CRS, S)$. Its efficiency can be achieved by reducing the input vector x^{j^*} to the point $D\ (\theta^*{}_1 x_1^{j^*}, \theta^*{}_2 x_2^{j^*})$. The output measures of technical efficiency can be similarly shown on the figure.

Similarly, both radial and non-radial measures of technical efficiency can be compared with respect to technology exhibiting variable returns to scale. The former measure of efficiency is computed from BCC model whereas the latter measure is from the Russell *(VRS)* model. These two models are very much different from their earlier counterparts, in that convexity constraints are relaxed. These two measures are also illustrated in the Figure 1. Here the variable returns to scale technology represented by the input *set* $L(y^j : VRS, S)$ is superimposed in the constant returns to scale technology, $L(y^j : CRS, S)$. This means that the *VRS* boundary is literally contained or nested within the *CRS* boundary. The radial measure *(VRS,S)* indicates the efficiency of x^{j^*} (denoted by the point P) in the production of u^{j^*} by contracting all the inputs equiproportionately to the point Q characterized by the input vector $(\theta^* x_1^{j^*}, \theta^* x_2^{j^*})$ whereas in the non-radial measure the efficiency is achieved by contracting the input vector x^{j^*} (P) non-proportionally to the point B $(\theta^*{}_1 x_1^{j^*}, \theta^*{}_2 x_2^{j^*})$ in the efficient subset of the production technology characterized by the facet AB. A general relationship governing the efficiency scores and the scale characteristics of the estimated boundary is apparent here. For a given performance, a nested technology implies a higher technical efficiency score than a non-nested counterpart. This is a qualitative ranking of technical efficiency of a unit, which is of important practical relevance.

2.4 A Comparison of Radial and Non-Radial Measure of Technical Efficiency

In general, the radial and the non-radial measure do not assign the same efficiency value to a given input vector $x^{j^*} \in L(y^j : CRS/VRS, S)$, but it is true that both measures give efficiency rating 1 for input vector x^{j^*} only when x^{j^*} belongs to the efficient subset $L^*(y^j : CRS/VRS, S)$ as defined in equation 5. It is due to the fact that $F_i(y^{j^*}, x^{j^*} : CRS/VRS, S)$ relates $x^{j^*} \in L(y^j : VRS, S)$ radially to an element of *isoquant* $\bar{L}(y^j : CRS/VRS, S)$, whereas $RM_i\ (y^{j^*}, x^{j^*} : CRS/VRS, S)$ relates $x^{j^*} \in L(y^{j^*} : CRS/VRS, S)$ nonradially to an element of efficient subset $L^*(y^j: CRS/VRS, S)$.

Both measures are graphically illustrated in Figure 1. Let us consider the input vector $x^{j^*} \in L(y^j : CRS/VRS,S)$ In this case, $RM_i(y^{j^*}, x^{j^*}: CRS,S)$ compares x^{j^*} with any point on the efficient subset of the input set, $\bar{L}(y^{j^*}: CRS,S)$, characterized by the line CD whereas $F_i(y^{j^*}, x^{j^*}: CRS,S)$ relates it with the point $(\theta^* x_1^{j^*}, \theta^* x_2^{j^*})$. But, for $x^{j^*} \in L(y^{j^*}: CRS,S)$, the radial measure compares x^{j^*} with $(\theta^* x_1^{j^*}, \theta^* x_2^{j^*})$ which belongs to the *isoquant*, $L(y^{j^*}: CRS/VRS,S)$, with some slack of input x_1. The Russell measure eliminates this slack by comparing x^{j^*} with $(\theta^*_1 x_1^{j^*}, \theta^*_2 x_2^{j^*})$ that belongs to the efficient subset of input set, $L^*(y^j : CRS,S)$). It is thus geometrically clear that, in general,

$$0 \leq RM_i(y^{j^*}, x^{j^*}: CRS/VRS,S) \leq F_i(y^{j^*}, x^{j^*}: CRS/VRS,S) \leq 1$$

A notable feature of the Farrell measure of technical efficiency is that it does not coincide with Koopmans' definition of technical efficiency. Koopmans' definition is stringent, requiring simultaneous membership in both efficient subsets, while the Farrell measure only requires membership in an *isoquant*. Thus Farrell measure correctly identifies all Koopmans-efficient producers as being technically efficient, and also identifies as being technically efficient any other producers located on an *isoquant* outside the cone spanned by the efficient subset.

The practical significance of this distinction depends on how many observations lie outside the efficient subset. However, the problem of distinguishing the two definition disappears in much econometric analysis, in which the parametric functional form used to represent production technology (e.g., Cobb-Douglas) imposes the assumption of equality between *isoquant* and efficient subsets, and eliminates slacks by assuming it away. But the problem is deemed significant in practice in data envelopment analysis approach and much effort has been put in the literature to suggest possible remedies. The first possible remedy is suggested in the additive model of Charnes *et al.* (1985) and latter in the extended additive model of Charnes *et al.* (1987). The non-radial multiplicative measure of Färe and Lovell (1978) is suggested as a possible remedy. This measure, however, has its own problems (Bol (1986, 1988), Russell (1985, 1988)). The most widely used measures (Charnes *et al.* (1978), Ali (1989, 1991)) that combine the Farrell radial measure and any remaining slacks into a single measure of technical efficiency have been proposed as a possible remedy. However, the various combination measures have flaws of their own (Boyd and Färe (1984), Färe and Hunsaker (1986)). It is to be noted here that each of the suggested measure has the great virtue of guaranteeing that an unit is efficient if and only if it belongs to the efficient subset of the production technology. Finally, if prices are available and economic efficiency can be calculated, then the suggested measure as a possible remedy is the measure of economic efficiency (either in the form of cost efficiency and/or revenue efficiency) where there is no such distinction between the definition and the measure of economic efficiency.

A problem which has often cropped up in the DEA literature is concerned with whether the efficiency of a unit (say, $DMU_{j\bullet}$) which is not in the region of the efficient subset of the production technology, is to be evaluated against the point $(\theta^*_1 x_1^*, \theta^*_2 x_2^*)$ or against the points on the linear extrapolation of the segment CD in the direction of the point D towards the input x_1 axis of the CCR efficient frontier (See Figure 1). The Russell measure of efficiency deals with the evaluation against the point $(\theta^*_1 x_1^*, \theta^*_2 x_2^*)$. Green *et al.* (1996) have discussed the process of evaluation of the DMUs (operating not on the region of the efficient technology) against the point on the linear extrapolation of the segment CD of the CCR efficient frontier.

2.5 Weighted Russell Measure of Efficiency

The potential problem of the Farrell measure arises when there exists slacks in some but not all of the inputs after radial efficiency is achieved. Färe and Lovell (1978) not only recognized this problem but provided a solution by introducing the nonradial Russell measure of efficiency. Also, an alternative method is suggested in the study of Ruggiero (1996). In the presence of slacks, rather than determining maximum radial reduction in all inputs holding output constant, the Russell measure minimizes the unweighted arithmetic mean of proportional reductions in all individual inputs. Let us consider the Figure 2.

INSERT FIGURE 2

DMU A is Farrell efficient, achieving $F_i = 1$. This results because DMU A is compared to itself in the solution of DEA model (8). The Russell measure, on the other hand, allows non-radial contraction of inputs and hence, compares A to B. Solution to nonradial model results in a Russell measure $RM_i = 0.83$. This solution is obtained from $\theta_1 = 1$ and $\theta_2 = 2/3$, i.e. A is efficient in the use of x_1 but inefficient in the use of x_2 relative to B. Using Russell measure, B and D are efficient, while A, C and E are not. It is to be noted here that Farrell and Russell measure of efficiency for DMU C are the same since no additional slacks exists after radial reduction is achieved. It is clear that the Russell measure appears to provide a solution to the problem of additional slacks that is inherent in the Farrell measure. This solution, however, requires an implicit assumption that all have the equal factor shares in the production process, i.e. all inputs equally effect the level of potential production. It is to be noted here that both Farrell measure of efficiency for DMUs A and E are the same. In this case, DMUs A and E are efficient in only one input and equally inefficient in the other. This lead to equivalent Russell measured for these two DMUs. As discussed earlier, A is compared to B in the solution of nonradial model. Likewise, E is evaluated relative to D, which uses the same amount of x_2 but two thirds as much of x_1.

The problem with the Russell measure is shown in Figure 2, where the true (but unknown) *isoquants* are superimposed on the piecewise linear *isoquant*. The true *isoquants* are generated from the production function $y = x_1^{3/4} x_2^{1/4}$. As is seen from Figure 2, while all DMUs produce the same level of output, the efficient amount of output differs for all DMUs. DMU B, which is Koopmans efficient, can produce the least amount of output (y^B) given its input usage. DMU E could have produced the most amount of output (y^E) given its high level of x_1. This is interesting because the Russell measure identifies DMUs A and E equally efficient and more efficient than DMU C. As shown, DMU C is more inefficient than DMU A but more efficient than DMU E. Consequently, both the Farrell and Russell measures fail to rank the DMUs properly.

The failure of the Russell measure can be attributed to the invalid assumption of equal weights when different inputs impact output differently in the production process. Ruggiero and Bretschneider (1998) extended the important model of Thanassoulis and Dyson (1992), which is motivated by the preferred input and output levels where each DMU can assign weights based on its preferences, to accommodate the excess slack inherent in the Farrell measure without assuming equal factor weights. Weights are not chosen to achieve preferred target levels but rather to recognize the possibility of differential factor weights in the production process. Weights are obtained from the empirical estimate of production function, through regression technique. The Weighted Russell measure can be computed from the following LP model of Ruggiero and Bretschneider (1998) as

$$WRM_i(y^{j*}, x^{j*} : VRS, S) = \min_{\lambda,\theta} \sum_{m=1}^{M} \omega_m \theta_m \quad \ldots\ldots(10)$$

$$s.t.\ y_n^{j*} \le \sum_{j=1}^{J} \lambda^{j*}{}_j y^j{}_n, n = 1,2,\ldots\ldots,N$$

$$\sum_{j=1}^{J} \lambda^{j*}{}_j x^j{}_m \le \theta_m x_m^{j*}, m = 1,2,\ldots\ldots,M$$

$$\sum_{j=1}^{J} \lambda_j^{j*} = 1$$

$$0 < \theta_m \le 1, \lambda_j \ge 0$$

This LP model results in a Weighted Russell measure of technical efficiency, where it is assumed that the weights are known *a priori*. The solution of this model does not necessarily result in a maximum score of unity. There are two ways, as suggested by Ruggiero and Bretschneider (1998), to convert the measure so that the maximum observed efficiency is unity: by adjusting the weights to sum to unity prior to measurement or by dividing the score from his model by the maximum observed efficiency.

2.6 Sensitivity Analysis

Since DEA is a data based approach, a study of sensitivity of efficiency is essential. Let us consider a DMU whose input-output vector is $(\theta^* x_1^{j*}, \theta^* x_2^{j*} - \varepsilon, y^{j*})$. Let it replace another DMU whose input-output vector is $(\theta^* x_1^{j*}, \theta^* x_2^{j*}, y^{j*})$. Here ε is an arbitrarily small positive quantity (See Figure 1). Now the *isoquant* $L(y^j : CRS, S)$ changes, and, consequently, the replacing DMU would be efficient. Thus, a slight change in the use of x_2, by an amount ε, attributed to a measurement error or noise (say), results in a change in the efficiency score. In general, DEA may remain robust to some perturbations in the data while it may be sensitive to others (Nunamaker, 1985).

In this paper we investigate the extent of change in the efficiency of extremely efficient (efficient in the Koopmans sense) DMUs due to change in their inputs. An efficient DMU is considered to be robust to a given increase in inputs if it remains efficient even after the change occurs in its input. This paper focuses on proportional increase in all inputs, since decrease in inputs cannot worsen an efficient DMU. The model by Zhu (1996) excludes the DMU_{j*} ($j^* \in J$) under evaluation while investigating the effect of such equiproportionate increase in all inputs of the DMU_{j*}. As Lewin and Minton (1986) point out, this kind of sensitivity analysis is important for establishing the robustness of efficiency scores. This model helps to find out the range over which DMU_{j*} remains efficient after the change in its inputs.

The Model:

The robustness model, discussed below, focuses on upward proportional variations of subset (S) of inputs. The maximum amount of change allowed in the input for DMU_{j*} is

$$\begin{aligned} \hat{x}_m^{\ j^*} &= d x_m^{\ j^*}, d \geq 1, m \in S \\ &= x_m^{\ j^*}, m \notin S \end{aligned} \qquad \text{......(11)}$$

Thus, the modified input-based CCR model[1] developed by Zhu (1996) is

$$\begin{aligned} &\min \rho \\ &s.t.\ \sum_{j=1, j\neq j^*}^{J} \lambda_j x_m^{\ j} \leq \rho x_m^{\ j^*}, m \in S \\ &\sum_{j=1, j\neq j^*}^{J} \lambda_j x_m^{\ j} \leq x_m^{\ j^*}, m \notin S \\ &\sum_{j=1, j\neq j^*}^{J} \lambda_j y_n^{\ j} \geq y_n^{\ j^*}, n = 1,2,\ldots\ldots, N \\ &\lambda_j, \rho \geq 0 \end{aligned} \qquad \text{......(12)}$$

The optimal solution to (12), denoted by ρ^*, determines the extent to which an efficient for DMU_{j*} remains efficient after the proportional increase (11) in inputs takes place.

3. The Empirical Application

3.1 The Data

This study uses data on inputs and outputs of 15 companies representing approximately the whole Indian Pig Iron and Sponge Iron industry for the year 1995. A single output (gross value added) and four inputs – raw

---

[1] This model is based on constant returns to scale since allowing variable returns to scale make the solutions infeasible, as pointed out by Zhu (1996)

materials, energy, labor and capital- are considered. All the data necessary for this study were taken from data package CIMM of the Centre for Monitoring Indian Economy, Bombay. The output, i.e., gross value added consists of wages, lease-rent, interest, depreciation, net-profit and tax. Since the number of workers is not available unit-wise, we have taken, for input, total wages and salaries for labor instead of a proxy for labour. The capital input is given by total depreciation plus profit relative to the total assets (i.e., fixed assets + working capital) employed. Thus, the capital input for any particular period is computed as the sum of depreciation for that period and current assets employed times the return on assets in the base period (Sumanth, 1985).

3.2 CCR, BCC and Russell (CRS & VRS) LP Model

In our case, $J = 15$, the number of firms/DMUs. The number of inputs $M = 4$, and the number of outputs $N = 1$. The decision variables are the shadow prices $\lambda_1, \lambda_2, \ldots, \lambda_{15}$, and the reduction factors are θ in the CCR and BCC model, and $\theta_1, \theta_2, \theta_3$ and θ_4 in the Russell CRS and VRS model. We are required to minimize the objective functions $F_i\ (y^{j*}, x^{j*} : CRS/VRS,S)$ (i.e., equation 8), and $RM_i\ (y^{j*}, x^{j*} : CRS/VRS,S)$ (i.e., equation 9) subject to the respective constraints. We have used the DEA software DEAP (version 3.1) as well as LP software LINDO to solve the LP models.

3.3 Results

3.3.1 Comparisons of Radial and Non-radial Measures of Efficiency

The input-output data are used to compute the Farrell and the Koopmans measure of technical efficiency. Table 1 gives the results of the Farrell's model.

INSERT Table 1

The first column of Table 1 gives the serial number of DMUs. The second column, representing θ, shows the efficiency level of the DMUs. Thus, DMU_1 has efficiency rating of 0.438. It implies that about 56% (= (1 - 0.438) *100) of the observed input could be reduced while maintaining the present level of output. The third column of Table-1 represents λ_j, and indicates the serial number of peer-group (reference-group) DMUs against whom the efficiency of DMU_j is compared. Another interpretation of λ_j's is that the DMU_j 's frontier output is a convex combination of the outputs of the peer-group DMUs against whom the efficiency of DMU_j is evaluated. In case of DMU_1, for example, the DMUs 2 and 8 constitute the peer-group. Similar interpretations also hold for the rest of the DMUs in the production technology. It is evident from this column that the peer group does not necessarily consist of only one DMU. So in the case of peer group consisting of more than one DMU, the problem could be one of choosing a single DMU in the peer group against whom the efficiency of a DMU under evaluation is judged. Here the absolute values of λ's of the respective DMUs in the peer group are used in deciding the single referent DMU. The largest value of λ

determines the DMU in the peer group with which the DMU under evaluation is the closest. Table-1 shows that DMU_1 has the single referent DMU_8 as the λ value ($\lambda^1_8 = 1.118$) corresponding to the unit 8 is the highest among all its peer-group DMUs. Similar interpretations can be given for the remaining DMUs. The input and output slacks are presented in the fourth column. Here we find that slacks present in case of all inefficient DMUs whereas they disappear for all efficient units. The ranking of the DMUs (represented in the last column) is based on the efficiency ratings in descending order of magnitude. It gives the relative performance position of the DMUs in the production technology of the Indian Pig Iron and Sponge Iron Industry in 1995.

The results of input-oriented model of CCR reported in Table 1, reveals that three out of 15 DMUs are efficient with their corresponding λ values as 1. These efficient units (2, 8 and 10) constitute the vertices of the frontier production technology. For the inefficient DMU_1, the DMUs 2 and 8 constitute the peer group. It means that the projected input-output vector of DMU_1 is a convex hull of the observed input-output vectors of DMUs 2 and 8. And its inefficiency is quantified by 56%, implying that 56% of its observed input vector could have been reduced to achieve the given output. Similar interpretations hold good for other inefficient DMUs. The units 2, 8 and 10 are on the referent plane for units 3, 4, 5, 6, 7, 9, 12, and 15. The units 2 and 10 constitute the peer group for units 11 and 14, and finally the unit 8 alone forms the peer group for the unit 13. In case of peer group consisting of more than one DMU, the single referent DMUs are 8 and 10 the DMUs 1, 3, 4, 5, 6, 7, 12, 13, 15, and 9, 11, and 14 under evaluation, respectively.

It is to be noted that a DMU may be efficient even if it has some input slack. Fortunately, our results show that there are no input slacks for these three efficient units. But, a closer examination of the efficiency estimates of all the inefficient units in the CCR model reveal that they are evaluated against the points on the *isoquant* characterized by a subset of the input set for each output. It is clearly evident from their projected input levels that they have each slack in their inputs when compared to their nearest point on the efficient subset of production technology. The relevant question that can be asked now is that had there been some DMUs operating on these projected points, they would have been efficient in the Farrell's sense. It is due to the fact that the radial measure of Farrell drives each input vector equiproportionately towards the *isoquant* of the input set for each output. The problem with the Farrell's measure is that it gives higher efficiency rating for both efficient and inefficient units which are not operating on the efficient subset of the production technology. In the Koopmans (CRS) model, however, this problem does not occur. The results of the Koopmans (CRS) model are reported in Table 2.

Unlike the earlier models, Russell's CRS model has four reduction factors θ_1 for input x_1, θ_2 for input x_2, θ_3 for input x_3 and θ_4 for input x_4 since it is non-radial in nature. These vectors show that for the unit j, inputs x_1 and x_2 could be reduced to $\theta_1 x_1$ and $\theta_2 x_2$ respectively without affecting its specified volume of production.

The other terms in Table 5.2 do not need any further interpretations.

INSERT Table 2

Table 2 reveals the effect of change in model orientation (from radial to non-radial). As seen in this table, only three units (2, 8, and 10) are on the frontier production technology and are thus efficient; and the remaining ones are inefficient. These three units were also efficient in the radial CCR model. As expected, for all the inefficient units which are not properly enveloped by the production technology, *L(y)*, the efficiency ratings have been drastically reduced compared to the CCR model. And also, the input and output slacks for all the units (both efficient and inefficient) are all zero, since the Koopmans measure relates a unit to a unit of the efficient subset of the *isoquant*.

Let us now turn to compare both radial and non-radial measures of efficiency with respect to technology exhibiting variable returns to scale. The results are reported in Table 3 and Table 4.

INSERT Table 3 and Table 4

In both these measures, six units (2, 5, 6, 8, 10, 13) are found to be efficient. As expected, the inefficient units in the radial BCC model have slacks in their input-output vectors whereas slacks disappear in the non-radial Koopmans (VRS) model. And for these inefficient units which are not properly enveloped by the production technology, the efficiency ratings in the Koopmans (VRS) model have been drastically gone down compared to the BCC model. This observation conforms to the general theoretical result:

$$RM_i\,(y^{j^*}, x^{j^*} : CRS/VRS, S) \leq F_i\,(y^{j^*}, x^{j^*} : CRS/VRS, S)$$

3.3.2 Comparisons of Radial and Non-radial Measures of Projected Inputs

Let us now consider the case where a DMU manager compares his current employment input and projected input levels. The projected input levels are the extent to which the current input levels could be reduced without rendering the continued production of output infeasible. It has a managerial decision-making orientation, geared towards the efficient utilization of resources. The Farrell's model is used to find out the projected input levels given in the Table 5, whereas Table 6 reports the results of the Russell's CRS model, which is used to estimate efficiency in the Koopmans sense. Table 5 shows that there is no difference between the projected and current level of inputs of DMUs 2, 8, and 10, as they are efficient. For the inefficient units, the projected input levels are less than those of their current values. In case of inefficient unit 1, the current employment levels of all four inputs are 33.67, 3.18, 3.25 and 28.94, whereas its projected levels are 14.75, 1.39, 1.42 and 12.68 respectively. The remaining DMUs can also be interpreted in a similar fashion.

INSERT Table 5

In case of Russell's CRS model, it is found from Table 6 that for the efficient units 2, 8, and 10, projected and current input levels are the same. But for the inefficient units, the projected input levels are no longer the same as their current values. A closer look at these two measures reveals that there is certainly a difference in estimates of projected input vectors. And this difference will be there so long as the *isoquant* is not equal to its efficient subset.

INSERT Table 6

Now let us compare the projected input vectors obtained from both radial and non-radial measures characterized by the VRS technology. These are reported in Table 7 and Table 8.

INSERT Table 7 and Table 8

The observations obtained from these two VRS models are similar to those from two CRS models.

3.3.3 Comparison of Radial and Non-radial Peer Groups

When a production technology in the competitive set up comprises a large number of DMUs operating at various scales, it would be difficult on the part of a particular inefficient DMU to look for its peer group DMUs against whom it will compare its efficiency. The DEA model helps to identify the reference groups for the inefficient units. Table 9 reports the DMUs along with their peer groups in both the Farrell and the Koopmans (CRS) sense. As seen from the Farrell measure, DMU_6 alone constitutes the peer group for DMU_{13}; DMUs 2, 8, and 10 form the peer group for DMUs 3, 4, 5, 6, 7, 9, 12, and 15; DMUs 2 and 10 are the peer group for DMUs 11 and 14; and DMUs 2 and 8 are the peer group for DMU_1. But the efficient DMUs 2, 8 and 10 are the peer groups for themselves since they are on the vertices of the frontier production technology. Among the peer groups consisting of more than one unit, the single referent DMU is mostly the unit 8 except for the inefficient units 9, 11 and 14. The single referent DMU, in case of DMUs 9, 11 and 14 is the unit 10.

INSERT Table 9 AND Table 10

The results for the Koopmans (CRS) model reveal that DMU_8 alone is the peer group for units 1, 3, 6, 7, 12, 13 and 15, whereas along with DMU_2, it constitutes the peer group for unit 4. And DMU_2 alone is the peer group for units 5, 9, 11 and 14. So far as the single referent DMUs of the peer groups are concerned, the

DMU_8 is found to be the only unit against whom the efficiency of the inefficient units are evaluated. Both the models agree on the same conclusion that the dominant peer unit 8 is the same for units 1, 3, 6, 7, 12, 13 and 15 whereas they vary substantially for the units 4, 5, 9, 11 and 14. Another interesting observation from the comparison of both these models is that the peer group size in the Koopmans (CRS) model is small as compared to the Farrell model.

The short-run comparison of peer groups (represented in Table 10) between BCC (VRS) and Koopmans (VRS) models leads us to make the similar observations as made in long-run comparison. Here the dominant peers in the BCC model are mostly the units 8 and 5 whereas it is only unit 8 in the Koopmans (VRS) model. The peer group size in the Koopmans (VRS) model is small compared to in the BCC model.

Given that the differences between the CRS and VRS results are appreciable, it is important to examine how this might affect the usefulness and interpretation of DEA in setting technical efficiency standard and targets of the firms in the Indian Pig Iron and Sponge Industry. In particular, some reconciliation of the large differences in efficiency status between two sets of results is required. One of the most useful arguments in this regard is the interpretation of DEA efficiency found in the work of Grosskopf and Valdmanis (1987). Essentially they argue that the CRS technology should be interpreted as reflecting long-run performance possibilities whereas the VRS assumption indicates feasible attainments in the short run. On the basis of the long-run CRS the volume of inputs required for given production will be smaller than those suggested by the short-run VRS technology. The CRS targets are effectively adjustments towards long-run equilibria, i.e., the minimum point of a U-shaped average cost curve. So, in the short run, cost incurred on the volume of inputs used will be greater than that in the long run.

The problem of the practitioners is not then in deciphering two seemingly contradictory sets of results on efficiency status, but in the initial choice of technology assumption. Once this is motivated, the nature of bias imparted to the efficiency measure is explicable *a priori*. In particular the CRS results can be taken as indicators of the proximity of the plants to a long-run notion of best practice. It follows that the finding of fewer examples of best practice (3 out of 15 plants) is to be expected from the CRS result because the long-run cost attainments are expected to be lower than those set by best practice in the short run. Analogously, a larger share of cross section (6 out of 15 plants) is efficient in the short run.

3.3.4 Weighted Russell Results

In order to measure efficiency using the Weighted Russell measure, it is necessary to provide the weights. As pointed out by Ruggiero and Bretschneider (1998), the existence of technical inefficiency could bias the parameter estimates because the independent variables are correlated with the error term. We therefore employed OLS regression to the sample of only Koopmans efficient units to estimate the production function. Although the results are not reported, OLS regression was run to the full data set but there was a difference in estimates compared to those obtained by applying OLS to only Koopmans efficient units. This difference in estimates certainly attributes to inefficiency.

INSERT TABLE 11

We employed four different functional forms to a sample of six Kopmans efficient units. The results are reported in Table 11. It was found in all the cases that the null hypothesis of CRS is no more valid. The Weighted Russell measure of efficiency was then calculated for each set of weights obtained from four different production functions. This measure of efficiency provides a summary measure of inefficiency that incorporates not only he excess input usage identified by the Farrell measure but also the additional slack that exists in individual inputs. That is why this measure provides a measure of Koopmans efficiency.

INSERT TABLE 12

Table 12 analyzes the performance of each measure based on three criteria, mean and standard deviation of the absolute difference between true and measured efficiency and the rank correlation of the measured and true efficiency. These performance measures were calculated for both the CRS and VRS DEA models and for each of four different production functions. The first measure, the mean of the absolute difference between the measured and true efficiency provides a measure of the proximity of the measure on average to the true efficiency. The standard deviation of this absolute difference provides a measure of variability within a sample. Lower standard deviations are preferred. The final measure, Spearman's rank correlation between measured and true efficiency, is perhaps the most important since it captures the ability to correctly rank observation. A high rank correlation suggests that the measure performs well in identifying differential inefficiency.

It is found from Table 12 that the Weighted Russell measure achieved the highest rank correlation of the measured and true efficiency for both DEA models in most of the cases. This was, however, expected from our above discussion. But the Farrell measure outperformed both Weighted Russell and Russell measures in all the cases using the mean absolute difference criterion and in most of the cases using standard deviation criterion. Under both CRS and VRS assumptions, the Farrell measure had a lower mean absolute difference and lower standard deviation relative to Russell and Weighted measures. This calls into question the advantage of using - Russell measure when input slacks exist and - Weighted Russell measure when the inputs have differential importance in the production process. This result may be due to the fact that our Farrell efficient units do not have slacks but the slacks remain there in the peers of the inefficient units identified by Farrell measure.

3.3.4 Results for Sensitivity Analysis

The results for both radial CCR and non-radial Russell measure under CRS agree on the one conclusion that only three units, 2, 8 and 10, are efficient in the Koopmans sense, though the results substantially vary in other respects (e.g., in respect of efficiency ratings and projected input-output vectors).

INSERT Table 13

Table 13 shows that DMU_2 is robust over input ranges from 292.43 to 676.073 of input x_1, from 89.03 to 205.83 of input x_2, from 6.20 to 14.334 of x_3 and from 178.68 to 413.093 of input x_4. DMU_8 remains efficient over input ranges from 12.90 to 26.434 of input x_1, from 1.16 to 2.377 of input x_2, from 0.53 to 1.086 of x_3 and from 10.78 to 22.09 of input x_4. Finally, DMU_{10} is robust over input ranges from 16.37 to 96.502 of input x_1, from 1.16 to 6.838 of input x_2, from 4.99 to 29.416 of x_3 and from 0.43 to 2.535 of input x_4. Here the index for input subset (S) for DMU_2, DMU_8 and DMU_{10} takes all the inputs into consideration, i.e., S = (1, 2, 3, 4). The implication is that each of the three DMUs preserves its efficiency within the stated increase in input range.

4. Summary

Two dominant views exist in the literature insofar as the measurement of technical efficiency of decision making units are concerned: (1) the non-radial measure (Russell CRS and VRS model) founded on the work of Koopmans, and (2) the radial measure (CCR and BCC model) based on the work of Farrell. In the current paper, we have discussed the theoretical underpinnings of these two measures and have applied them to the real-life data of Indian Pig Iron and Sponge Iron Industry. We have used the framework of Data Envelopment Analysis for this purpose.

The results of the empirical study may be summarized as under:

1. As expected,

   (i) the radial measures give higher efficiency ratings for the DMUs which are not properly enveloped; and

   (ii) the projected input levels for the inefficient units are less than their current values for both the models.

2. The efficient units are the same for both measures in the short run as well as in the long run. This, however, is a coincidence, because the efficient units in the radial measures had zero slack.

3. Both the measures of efficiency are clearly distinguished in case of all the inefficient units

where their projected input levels contain input slacks in the radial measures, but, in the non-radial measures, these slacks disappear.

4. The peer groups for the inefficient units are substantially different for both the measures.

5. In both the measures, DMU_8 stands out as the most dominant (referent) peer among the efficient units in the long run. In the short run DMUs 8 and 5 are the most dominant peer in the radial measure whereas DMU_8 alone appears dominant in the non-radial measure.

An examination of the value of output (= 18.87 million rupees) and in the input levels (raw-materials = 12.90 million rupees, energy = 1.16 million rupees, wages = 0.53 million rupees and capital = 10.78 million rupees) for DMU_8 indicates that these values are comparable to the range of output and input values of the inefficient units.

6. Peer group size has reduced, indicating more `precise' result for the non-radial measure.

7. The ranking of the inefficient units substantially differed in both the measures.

8. The sensitivity analysis carried out under the assumption of CRS on the two models indicates that only three units, 2, 8 and 11, are efficient, and that they are robust. The number of efficient units has considerably come down because of the relaxed assumption of CRS. An examination of the input and output levels of these three units indicates that these values are very much comparable to the values of the other units.

9. The highest rank correlation obtained from this new measure suggests that this model should be considered as an alternative DEA model when slack exists and inputs have differential importance in the production process.

Admittedly, the number of units considered in this study is small (only fifteen). Although the results were not unexpected, we feel that with a larger number of units, we could have got more interesting results. For example, our results could have shown that a few efficient units in the Farrell's sense turn out to be inefficient in the Russell's sense; or that certain efficient units are not robust in the face of input perturbations.

References

1. Ali, A. I., 1989, IDEAS: Integrated data envelopment analysis system, Technical Report, Department of General Business and Finance, University of Massachusetts, Amherst, Mass.

2. Ali, A. I., 1991, Data envelopment analysis: a unique perspective, Working Paper, School of Management, University of Massachusetts, Amherst, Mass.

3. Banker, R. D., A. Charnes and W. W. Cooper, 1984, Some models for estimating technical and scale inefficiencies in data envelopment analysis, Management Science 30, 1078-1092.

4. Bol, G., 1986, On technical efficiency measures: a remark, Journal of Economic Theory 38, 380-385.

5. Bol, G., 1988, On the definition of efficiency measures: a note, in: W. Eichhorn, ed., Measurement in economics: theory and applications of economic indices (Physica-Verlag, heidelberg).

6. Boyd, G. and R. Färe, 1984, Measuring the efficiency of decision making units: a comment, European Journal of Operational Research 15, 331-332.

7. Chang, K.-P. and Y.-Y Guh, 1991, Linear production function and the data envelopment analysis, European Journal of Operational Research 52, 215-223.

8. Charnes, A., W. W. Cooper, B. Golany, L. M. Seiford and J. Stutz, 1985, Foundations of data envelopment analysis for pareto-koopmans efficient empirical production functions, Journal of Econometrics 30, 91-107.

9. Charnes, A., W. W. Cooper and E. Rhodes, 1978, Measuring the efficiency of decision making units, European Journal of Operational Research 2, 429-444.

10. Charnes, A., W. W. Cooper and E. Rhodes, 1979, Sort communication: measuring the efficiency of decision making units, European Journal of Operational Research 3, 339.

11. Charnes, A., W. W. Cooper, J. Rousseau and J. Semple, 1987, Data envelopment analysis and axiomatic notions of efficiency and reference set, Research Report CCS 558, Center For Cybernetic Studies, The University of Texas, Austin, TX 78712.

12. Charnes, A., J. J. Rousseau and J. Semple, 1992, Non-archimedean infinitesimals, transcendentals and categorical inputs in linear programming and data envelopment analysis, International Journal of Systems Science 23, 2401-2406.

13. Debreu, G., 1951, The coefficient of resource utilization, Econometrica 19, 273-292.

14. Färe, R. and C. A. K. Lovell, 1978, Measuring the technical efficiency of production, Journal of Economic Theory 19, 150-162.

15. Färe, R., S. Grosskopf and C. A. K. Lovell, 1994, Production frontiers (Cambridge University Press, Cambridge).

16. Färe, R. and W. Hunsaker, 1986, Notions of efficiency and their reference sets, Management Science 32, 237-243.

17. Farrell, M. J., 1957, The measurement of productive efficiency, Journal of Royal Statistical Society (A, General) 120, 253-281.

18. Green, R. H., J. R. Doyle and W. D. Cook, 1996, Efficiency bounds in data envelopment analysis, European Journal of Operational Research 89, 482-490.

19. Grosskopf, S. and V. Valdmanis, 1987, Measuring hospital performance: a non-parametric approach, Journal of Health Economics 6, 89-107.

20. Koopmans, T. C., 1951, Activity analysis of production and allocation (Wiley, New York).

21. Lewin, A. Y. and J. W. Minton, 1986, Determining organizational effectiveness: another look, and agenda for future research, Management Science 32, 514-538.

22. Lovell, C. A. K., 1993, Production frontiers and productive efficiency, in: H. O. Fried, C. A. K. Lovell and S. S. Schimdt, eds., The measurement of productive efficiency (Oxford University Press, Cambridge).

23. Morey, R., D. Fine and S. Loree, 1990, Comparing the allocative efficiencies of hospitals, Omega: The International Journal of Management Science 18, 71-83.

24. Nunamaker, T. R., 1985, Using data envelopment analysis to measure the efficiency of non-profit organizations: a critical evaluation, Managerial and Decision Economics 6, 50-58.

25. Ruggiero, J., 1996, Measuring technical efficiency in the public sector: An analysis of educational production, Review of Economics and Statistics 78, 499-509.

26. Ruggiero, J. and S. Bretschneider, 1998, The weighted Russell measure of technical efficiency, European Journal of Operational Research 108, 438-451.

27. Russell, R., 1985, Measures of technical efficiency, Journal of Economic Theory 35, 109-126.

28. Russell, R., 1988, On the axiomatic approach to the measurement of technical efficiency, in: W. Eichhorn, ed., Measurement in economics: theory and applications of economic indices (Physica-Verlag, Heidelberg).

29. Russell, R., 1990, Continuity of measures of technical efficiency, Journal of Economic Theory 51, 255-267.

30. Sumanth, D. J., 1985, Productivity engineering and management (McGraw-Hill Book Company, New York).

31. Thanassoulis, E. and R. G. Dyson, 1992, Estimating preferred target input-out levels using data envelopment analysis 56, 80-97.

32. Zhu, J., 1996, Robustness of the efficient DMUs in data envelopment analysis, European Journal of Operational Research 90, 451-460.

Table 1: Results of the Farrell (CRS) Measure of Technical Efficiency

DMUs	$F_i(y^{j^*}, x^{j^*} : CRS, S)$ (θ)	λ^j	Input & Output Slacks (x_1, x_2, x_3, x_4, y)	Ranking
1	0.438	$\lambda^1_2 = 0.001$ $\lambda^1_8 = 1.118$	0, 0, 0.823, 0.421, 0	12
2	1.000	$\lambda^2_2 = 1.000$	0, 0, 0, 0, 0	1
3	0.613	$\lambda^3_2 = 0.002$ $\lambda^3_8 = 0.886$ $\lambda^3_{10} = 0.186$	1.169, 0, 0, 0, 0	8
4	0.780	$\lambda^4_2 = 0.024$ $\lambda^4_8 = 0.349$ $\lambda^4_{10} = 0.145$	26.378, 0, 0, 0; 0	3
5	0.817	$\lambda^5_2 = 0.009$ $\lambda^5_8 = 0.092$ $\lambda^5_{10} = 0.074$	6.181, 0, 0, 0, 0	2
6	0.658	$\lambda^6_2 = 0.000$ $\lambda^6_8 = 0.521$ $\lambda^6_{10} = 0.034$	9.229, 0, 0, 0, 0	7
7	0.714	$\lambda^7_2 = 0.001$ $\lambda^7_8 = 0.618$ $\lambda^7_{10} = 0.106$	2.480, 0, 0, 0, 0	5
8	1.000	$\lambda^8_8 = 1.000$	0, 0, 0, 0, 0	1
9	0.658	$\lambda^9_2 = 0.014$ $\lambda^9_8 = 0.170$ $\lambda^9_{10} = 0.405$	0, 0, 1.183, 0, 0	7
10	1.000	$\lambda^{10}_{10} = 1.000$	0, 0, 0, 0, 0	1
11	0.542	$\lambda^{11}_2 = 0.217$ $\lambda^{11}_{10} = 0.652$	6.834, 13.507, 0, 0, 0	9
12	0.756	$\lambda^{12}_2 = 0.011$ $\lambda^{12}_8 = 0.417$ $\lambda^{12}_{10} = 0.120$	14.151, 0, 0, 0, 0	4
13	0.697	$\lambda^{13}_8 = 0.475$	44.228, 0, 1.366, 1.491, 0	6
14	0.441	$\lambda^{14}_2 = 0.011$ $\lambda^{14}_{10} = 0.159$	8.299, 0.034, 0, 0, 0	11
15	0.498	$\lambda^{15}_2 = 0.006$ $\lambda^{15}_8 = 0.487$ $\lambda^{15}_{10} = 0.210$	1.086, 0, 0, 0, 0	10

Table 2: Results of the Koopmans (CRS) Measure of Technical Efficiency

DMUs	$RM_i (y^{j^*}, x^{j^*} : CRS, S)$ $(\theta_1, \theta_2, \theta_3, \theta_4)$	λ^j	Input and Output Slacks (x_1, x_2, x_3, x_4, y)	Ranking
1	0.368 (0.439,0.418,0.187,0.427)	$\lambda^1_8 = 1.147$	0, 0, 0, 0, 0	10
2	1.000 (1.000,1.000,1.000,1.000)	$\lambda^2_2 = 1.000$	0, 0, 0, 0, 0	1
3	0.475 (0.494,0.502,0.234,0.670)	$\lambda^3_8 = 1.017$	0, 0, 0, 0, 0	7
4	0.516 (0.219,1.000,0.177,0.669)	$\lambda^4_2 = 0.039$ $\lambda^4_8 = 0.002$	0, 0, 0, 0, 0	4
5	0.539 (0.282,0.991,0.140,0.745)	$\lambda^5_2 = 0.013$	0, 0, 0, 0, 0	3
6	0.498 (0.278,0.617,0.423,0.677)	$\lambda^6_8 = 0.543$	0, 0, 0, 0, 0	6
7	0.546 (0.506,0.609,0.301,0.767)	$\lambda^7_8 = 0.687$	0, 0, 0, 0, 0	2
8	1.000 (1.000,1.000,1.000,1.000)	$\lambda^8_8 = 1.000$	0, 0, 0, 0, 0	1
9	0.474 (0.392,0.792,0.032,0.678)	$\lambda^9_2 = 0.027$	0, 0, 0, 0, 0	8
10	1.000 (1.000,1.000,1.000,1.000)	$\lambda^{10}_{10} = 1.000$	0, 0, 0, 0, 0	1
11	0.374 (0.443,0.325,0.165,0.561)	$\lambda^{11}_2 = .226$	0, 0, 0, 0, 0	9
12	0.504 (0.303,0.413,0.347,0.955)	$\lambda^{12}_8 = 0.772$	0, 0, 0, 0, 0	5
13	0.358 (0.085,0.697,0.108,0.540)	$\lambda^{13}_8 = 0.475$	0, 0, 0, 0, 0	12
14	0.278 (0.123,0.434,0.043,0.511)	$\lambda^{14}_2 = 0.014$	0, 0, 0, 0, 0	13
15	0.360 (0.373,0.314,0.143,0.611)	$\lambda^{15}_8 = 0.730$	0, 0, 0, 0, 0	11

Table 3: Results of the BCC (VRS) Measure of Technical Efficiency

DMUs	$F_i (y^{j^*}, x^{j^*} : VRS, S)$ (θ)	λ^j	Input & Output Slacks (x_1, x_2, x_3, x_4, y)	Ranking
1	0.521	$\lambda^1_2 = 0.006$ $\lambda^1_8 = 0.994$	3.050, 0, 1.130, 3.337, 0	9
2	1.000	$\lambda^2_2 = 1.000$	0, 0, 0, 0, 0	1
3	0.621	$\lambda^3_2 = 0.003$ $\lambda^3_8 = 0.881$ $\lambda^3_{10} = 0.115$	2.244, 0, 0.364, 0, 0	6
4	0.560	$\lambda^4_2 = 0.018$ $\lambda^4_5 = 0.586$ $\lambda^4_8 = 0.298$ $\lambda^4_{10} = 0.098$	23.123, 0, 0, 0, 0	3
5	1.000	$\lambda^5_5 = 1.000$	0, 0, 0, 0, 0	1
6	1.000	$\lambda^6_6 = 1.000$	0, 0, 0, 0, 0	1
7	0.871	$\lambda^7_5 = 0.261$ $\lambda^7_6 = 0.176$ $\lambda^7_8 = 0.563$	0, 0, 0.484, 0, 1.026	2
8	1.000	$\lambda^8_8 = 1.000$	0, 0, 0, 0, 0	1
9	0.776	$\lambda^9_2 = 0.002$ $\lambda^9_8 = 0.445$ $\lambda^9_{10} = 0.552$	0, 0.958, 0.991, 0, 0	5
10	1.000	$\lambda^{10}_{10} = 1.000$	0, 0, 0, 0, 0	1
11	0.544	$\lambda^{11}_2 = 0.215$ $\lambda^{11}_5 = 0.144$ $\lambda^{11}_{10} = 0.641$	5.852, 13.625, 0, 0, 0	8
12	0.797	$\lambda^{12}_2 = 0.006$ $\lambda^{12}_5 = 0.536$ $\lambda^{12}_8 = 0.380$ $\lambda^{12}_{10} = 0.078$	10.877, 0, 0, 0, 0	4
13	1.000	$\lambda^{13}_8 = 1.000$	0, 0, 0, 0, 0	1
14	0.598	$\lambda^{14}_2 = 0.000$ $\lambda^{14}_5 = 0.866$ $\lambda^{14}_{10} = 0.134$	5.239, 0.461, 0, 0, 0	7
15	0.544	$\lambda^{15}_5 = 0.237$ $\lambda^{15}_8 = 0.573$ $\lambda^{15}_{10} = 0.190$	0, 0.303, 0.079, 0, 0	8

Table 4: Results of the Koopmans (VRS) Measure of Technical Efficiency

DMUs	RM_i (y^{j^*}, x^{j^*} : VRS, S) ($\theta_1, \theta_2, \theta_3, \theta_4$)	λ^j	Input and Output Slacks (x_1, x_2, x_3, x_4, y)	Ranking
1	0.382 (0.430,0.521,0.173,0.405)	λ^1_2 =0.006 λ^1_8 =0.994	0, 0, 0, 0, 0	10
2	1.000 (1.000,1.000,1.000,1.000)	λ^2_2 = 1.000	0, 0, 0, 0, 0	1
3	0.477 (0.493,0.519,0.232,0.666)	λ^3_2 =0.001 λ^3_8 =0.999	0, 0, 0, 0, 0	6
4	0.560 (0.291,0.518,0.430,1.000)	λ^4_2 = 0.007 λ^4_5 = 0.215	0, 0, 0, 0, 0	5
5	1.000 (1.000,1.000,1.000,1.000)	λ^5_5 = 1.000	0, 0, 0, 0, 0	1
6	1.000 (1.000,1.000,1.000,1.000)	λ^6_6 = 1.000	0, 0, 0, 0, 0	1
7	0.710 (0.756,0.893,0.458,0.733)	λ^7_5 =0.485 λ^7_8 =0.515	0, 0, 0, 0, 0	2
8	1.000 (1.000,1.000,1.000,1.000)	λ^8_8 =1.000	0, 0, 0, 0, 0	1
9	0.576 (0.682,0.387,0.236,1.000)	λ^9_5 =0.284 λ^9_8 =0.566 λ^9_{10}=0.150	0, 0, 0, 0, 0	4
10	1.000 (1.000,1.000,1.000,1.000)	λ^{10}_{10} =1.000	0, 0, 0, 0, 0	1
11	0.388 (0.454,0.297,0.194,0.608)	λ^{11}_2 =0.196 λ^{11}_8 =0.804	0, 0, 0, 0, 0	9
12	0.582 (0.400,0.538,0.464,0.927)	λ^{12}_5 =0.353 λ^{12}_8 =0.647	0, 0, 0, 0, 0	3
13	1.000 (1.000,1.000,1.000,1.000)	λ^{13}_{13} =1.000	0, 0, 0, 0, 0	1
14	0.459 (0.422,0.424,0.295,0.694)	λ^{14}_5 =0.982 λ^{14}_8 =0.018	0, 0, 0, 0, 0	7
15	0.437 (0.523,0.433,0.204,0.589)	λ^{15}_5 =0.418 λ^{15}_8 =0.582	0, 0, 0, 0, 0	8

Table 5: A Comparison Between Present and Projected Input Levels Using Farrell (CRS) Model

DMUs	CI	PI	CI	PI	CI	PI	CI	PI
1	33.67	14.75	3.18	1.39	3.25	1.42	28.94	12.68
2	292.43	292.43	89.03	89.03	6.20	6.20	178.68	178.68
3	26.57	16.29	2.35	1.44	2.30	1.41	16.36	10.03
4	51.48	40.15	3.43	2.68	1.35	1.05	10.31	8.04
5	13.61	11.12	1.18	0.96	0.58	0.47	3.15	2.57
6	25.21	16.59	1.02	0.67	0.68	0.45	8.64	5.69
7	17.51	12.50	1.31	0.94	1.21	0.86	9.66	6.90
8	12.9	12.9	1.16	1.16	0.53	0.53	10.78	10.78
9	19.98	13.15	3.01	1.98	5.15	3.39	7.06	4.65
10	16.37	16.37	1.16	1.16	4.99	4.99	0.43	0.43
11	149.23	80.88	61.91	33.56	8.48	4.60	71.97	39.01
12	32.87	24.85	2.17	1.64	1.18	0.89	8.72	6.59
13	72.22	50.34	0.79	0.55	2.32	1.62	9.48	6.61
14	32.25	14.22	2.78	1.23	1.96	0.86	4.74	2.09
15	25.25	12.58	2.70	1.35	2.70	1.35	12.89	6.42

Table 6: A Comparison Between Present and Projected Input Levels Using Koopmans (CRS) Model

DMUs	CI	PI	CI	PI	CI	PI	CI	PI
1	33.67	14.79	3.18	1.33	3.25	0.61	28.94	12.36
2	292.43	292.43	89.03	89.03	6.20	6.20	178.68	178.68
3	26.57	13.13	2.35	1.18	2.30	0.54	16.36	10.97
4	51.48	11.28	3.43	3.43	1.35	0.24	10.31	6.90
5	13.61	3.84	1.18	1.17	0.58	0.08	3.15	2.35
6	25.21	7.00	1.02	0.63	0.68	0.29	8.64	5.85
7	17.51	8.87	1.31	0.80	1.21	0.36	9.66	7.41
8	12.9	12.9	1.16	1.16	0.53	0.53	10.78	10.78
9	19.98	7.83	3.01	2.39	5.15	0.17	7.06	4.79
10	16.37	16.37	1.16	1.16	4.99	4.99	0.43	0.43
11	149.23	66.11	61.91	20.13	8.48	1.40	71.97	40.39
12	32.87	9.96	2.17	0.90	1.18	0.41	8.72	8.32
13	72.22	6.13	0.79	0.55	2.32	0.25	9.48	5.12
14	32.25	3.97	2.78	1.21	1.96	0.08	4.74	2.42
15	25.25	9.42	2.70	0.85	2.70	0.39	12.89	7.87

Table 7: A Comparison Between Present and Projected Input Levels Using BCC (VRS) Model

DMUs	CI	PI	CI	PI	CI	PI	CI	PI
1	33.67	17.54	3.18	1.66	3.25	1.69	28.94	15.08
2	292.43	292.43	89.03	89.03	6.20	6.20	178.68	178.68
3	26.57	16.50	2.35	1.46	2.30	1.43	16.36	10.16
4	51.48	41.96	3.43	2.80	1.35	1.10	10.31	8.40
5	13.61	13.61	1.18	1.18	0.58	0.58	3.15	3.15
6	25.21	25.21	1.02	1.02	0.68	0.68	8.64	8.64
7	17.51	15.25	1.31	1.14	1.21	1.05	9.66	8.41
8	12.9	12.9	1.16	1.16	0.53	0.53	10.78	10.78
9	19.98	15.50	3.01	2.34	5.15	4.00	7.06	5.48
10	16.37	16.37	1.16	1.16	4.99	4.99	0.43	0.43
11	149.23	81.18	61.91	33.68	8.48	4.61	71.97	39.15
12	32.87	26.20	2.17	1.73	1.18	0.94	8.72	6.95
13	72.22	72.22	0.79	0.79	2.32	2.32	9.48	9.48
14	32.25	19.29	2.78	1.66	1.96	1.17	4.74	2.84
15	25.25	13.74	2.70	1.47	2.70	1.47	12.89	7.01

Table 8: A Comparison Between Present and Projected Input Levels Using Koopmans (VRS) Model

DMUs	CI	PI	CI	PI	CI	PI	CI	PI
1	33.67	14.48	3.18	1.66	3.25	0.56	28.94	11.73
2	292.43	292.43	89.03	89.03	6.20	6.20	178.68	178.68
3	26.57	13.09	2.35	1.22	2.30	0.53	16.36	10.89
4	51.48	15.00	3.43	1.78	1.35	0.58	10.31	10.31
5	13.61	13.61	1.18	1.18	0.58	0.58	3.15	3.15
6	25.21	25.21	1.02	1.02	0.68	0.68	8.64	8.64
7	17.51	13.24	1.31	1.17	1.21	0.55	9.66	7.08
8	12.90	12.90	1.16	1.16	0.53	0.53	10.78	10.78
9	19.98	13.62	3.01	1.17	5.15	1.21	7.06	7.06
10	16.37	16.37	1.16	1.16	4.99	4.99	0.43	0.43
11	149.23	67.79	61.91	18.41	8.48	1.64	71.97	43.75
12	32.87	13.15	2.17	1.17	1.18	0.55	8.72	8.08
13	72.22	72.22	0.79	0.79	2.32	2.32	9.48	9.48
14	32.25	13.60	2.78	1.18	1.96	0.58	4.74	3.29
15	25.25	13.20	2.70	1.17	2.70	0.55	12.89	7.59

Table 9: A Comparison of Peer Groups Between Farrell (CRS) and Koopmans' (CRS) Model w.r.t DMUs Under Evaluation

DMUs	Farrell (CRS) Model		Koopmans (CRS) Model	
	Peer Group	Dominant Peer Group	Peer Group	Dominant Peer Group
1	2, 8	8	8	8
2	2	2	2	2
3	2, 8, 10	8	8	8
4	2, 8, 10	8	2, 8	2
5	2, 8, 10	8	2	2
6	2, 8, 10	8	8	8
7	2, 8, 10	8	8	8
8	8	8	8	8
9	2, 8, 10	10	2	2
10	10	10	10	10
11	2, 10	10	2	2
12	2, 8, 10	8	8	8
13	8	8	8	8
14	2, 10	10	2	2
15	2, 8, 10	8	8	8

Table 10: A Comparison of Peer Groups Between BCC (VRS) and Koopmans (VRS) Model w.r.t DMUs Under Evaluation

DMUs	BCC (VRS) Model		Koopmans (VRS) Model	
	Peer Group	Dominant Peer Group	Peer Group	Dominant Peer Group
1	2, 8	8	2, 8	8
2	2	2	2	2
3	2, 8, 10	8	2, 8	8
4	2, 5, 8, 10	5	2, 5, 8	8
5	5	5	5	5
6	6	6	6	6
7	5, 6, 8	8	5, 8	8
8	8	8	8	8
9	2, 8, 10	10	5, 8, 10	8
10	10	10	10	10
11	2, 5, 10	10	2, 8	8
12	2, 5, 8, 10	5	5, 8	8
13	13	13	13	13
14	2, 5, 10	5	5, 8	5
15	5, 8, 10	8	5, 8	8

Table 11: Production Function Estimation

1) Linear Production Function With No Constant
$y = -0.13617\, x_1 + 3.3245\, x_2 + 1.0222\, x_3 + 1.3870\, x_4$
$R^2 = 1.0000,\ \bar{R}^2 = 0.9999$

2) Linear Production Function With Constant
$y = -1.2097 - 0.14279\, x_1 + 3.0532\, x_2 + 1.3436\, x_3 + 1.5287\, x_4$
$R^2 = 1.0000,\ \bar{R}^2 = 0.9998$

3) C-D Production Function With No Constant
$\ln y = 0.79576 \ln x_1 + 0.76013 \ln x_2 - 0.50603 \ln x_3 - 0.14181 \ln x_4$
$R^2 = 0.9246,\ \bar{R}2 = 0.8115$

4) C-D Production Function With constant
$\ln y = 4.0337 - 1.0136 \ln x_1 + 0.44341 \ln x_2 + 0.87759 \ln x_3 + 0.84185 \ln x_4$
$R^2 = 0.9993,\ \bar{R}^2 = 0.9965$

Table 12: Performance of Each Measure Based on Three Criteria

DEA RTS	Method	Absolute Difference Mean	Absolute Difference Std. Dev.	Correlation
Case I: Linear production function with no constant				
CRS	Weighted Russell	0.153	0.139	0.793
	Russell	0.230	0.203	0.593
	Farrell	0.101	0.099	0.861
VRS	Weighted Russell	0.099	0.105	0.921
	Russell	0.109	0.121	0.896
	Farrell	0.087	0.060	0.914
Case II: Linear production function with constant				
CRS	Weighted Russell	0.176	0.156	0.807
	Russell	0.241	0.219	0.611
	Farrell	0.116	0.109	0.861
VRS	Weighted Russell	0.106	0.108	0.929
	Russell	0.112	0.128	0.889
	Farrell	0.091	0.064	0.918
Case III: C-D production function with no constant				
CRS	Weighted Russell	0.376	0.227	0.486
	Russell	0.475	0.232	0.404
	Farrell	0.321	0.236	0.171
VRS	Weighted Russell	0.344	0.215	0.471
	Russell	0.371	0.255	0.154
	Farrell	0.270	0.219	0.089
Case IV: C-D production function with constant				
CRS	Weighted Russell	0.201	0.143	0.789
	Russell	0.298	0.206	0.618
	Farrell	0.155	0.114	0.746
VRS	Weighted Russell	0.142	0.152	0.832
	Russell	0.152	0.165	0.629
	Farrell	0.091	0.081	0.718

Table 13: Results of Sensitivity Analysis

DMU	ρ^*	x_1	x_1	x_2	x_2	x_3	x_3	x_4	x_4
2	2.311915	292.43	676.073	89.03	205.83	6.20	14.334	178.68	413.093
8	2.049137	12.90	26.434	1.16	2.377	0.53	1.086	10.78	22.090
10	5.895035	16.37	96.502	1.16	6.838	4.99	29.416	0.43	2.535

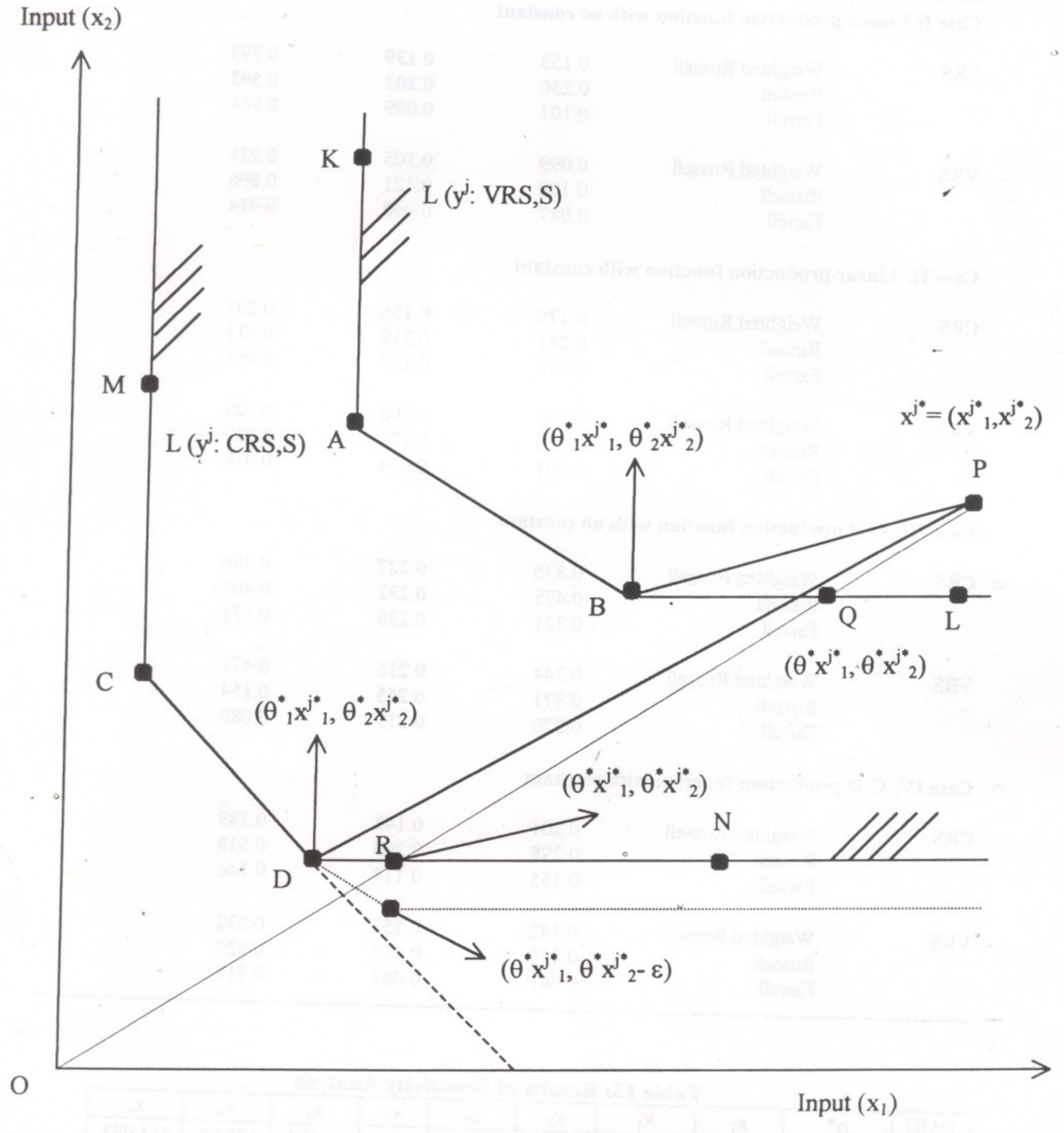


Fig. 1: Radial and Non-radial measures of technical efficiency under CRS and VRS technology

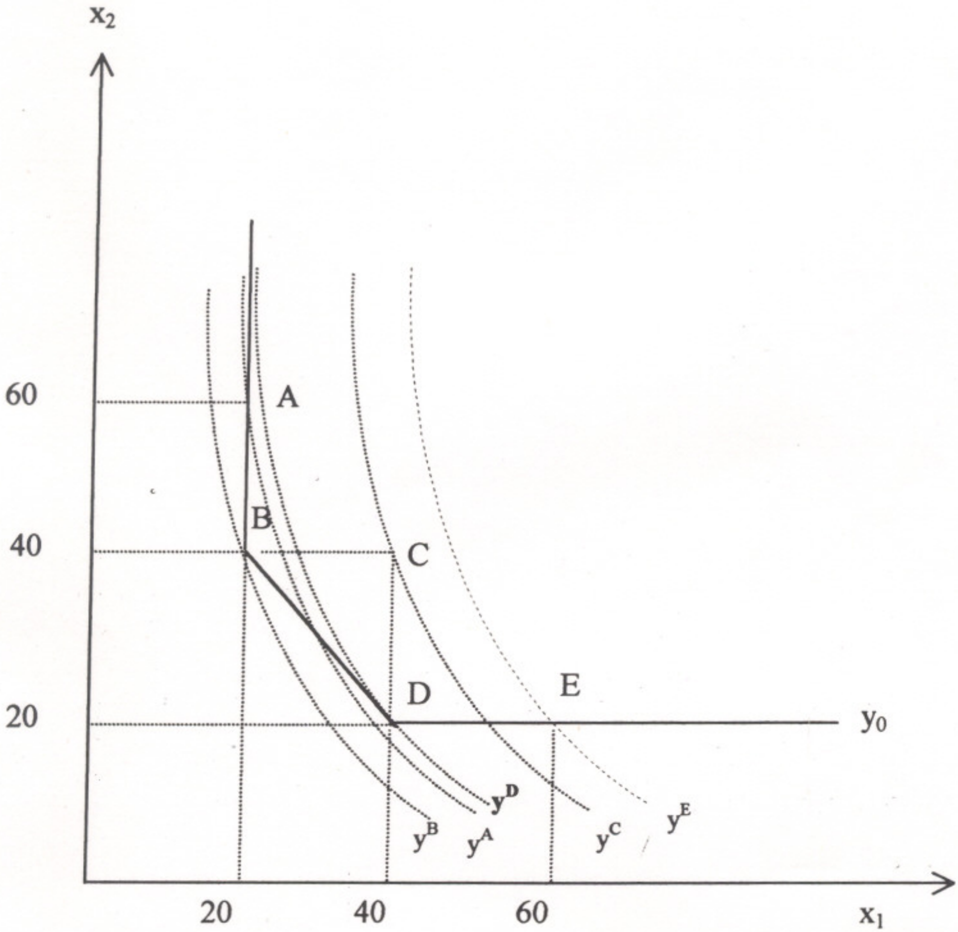


Fig. 2: Piecewise linear isoquant and True but unknown isoquant