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Abstract: This paper discusses the strengths and weaknesses of Best-Worst method (BWM) and Preference Ranking Optimization METHod for Enrichment Evaluation (PROMETHEE). Based on the discussion an attempt is made to amalgamate these two prominent MCDM methods in improving operational efficiency. BWM and PROMETHEE are integrated to bring in a number of BWM features into PROMETHEE including determination of weights. This amalgamation is used further to demonstrate reduction of number of pair-wise comparisons with respect to Analytic Hierarchy Process (AHP) with improved consistency. The integration is further validated through the evaluation of seven polytechnic colleges in an around Durgapur, the second largest educational hub of West Bengal.

Keywords: *BWM, MCDM, PROMETHEE, AHP*

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1. Introduction

Multiple criteria decision making (MCDM) is a sub discipline of Operations research which considers multiple criteria in real life decision making situations. Apart from the basic financial and elementary quantitative methods like cost benefit analysis, pros and cons analysis, maximin and minimax methods and lexicographic methods that have been developed to facilitate decision making (Linkov et al., 2004), there are two major operations research techniques namely multi-attribute decision making (MADM) and multi objective decision making (MODM) which contributes to MCDM largely (Rao, 2007).

Within this MADM family, Analytical Hierarchy Process (AHP) has been extensively used by different researchers in various fields of study because of its ease and ability to capture both subjective and objective parameters (Vaidya et al., 2006). This paper discusses two very prominent methods in MADM. The first one is a novel MADM method called Best Worst Method (BWM) developed recently by Rezaei (2015) and used by researchers in different fields of study (Chitsaz & Azarnivand, 2016; Mou et al., 2016; Ren & Chan, 2016; Rezaei et al., 2016; Ghaffari et al., 2017; Guo & Zhao, 2017). The other well-known method is Preference Ranking Optimization METHod for Enrichment Evaluation (PROMETHEE), a European outranking method developed by Brans (1982) and further improved by Brans and Vincke (1985). This method was also extensively used by global researchers in various field of decision making (Walther et al., 2008; Frikha et al., 2011; Athawale et al., 2012).

Section 2 discusses both the methods briefly. In section 3, comparative analysis of both the methods is presented. Section 4 discusses the inclusion of properties of BWM into PROMETHEE. Subsequently BWM-PROMETHEE integrated method is proposed. Section 5 compares this BWM-PROMETHEE method with AHP-PROMETHEE synergy proposed by Macharis et al (2004). Section 6 demonstrates an application of this novel integrated method in evaluating 7 polytechnic colleges from India. Conclusion, limitation and future scope of research is presented in Section 7.

2. An overview of BWM and PROMETHEE

2.1 Best Worst Method (BWM)

Best Worst method was introduced by Rezaei (2015) which demonstrates a potential in reducing the complexity and volume of calculation in comparison to prominent MCDM methods like Analytic Hierarchy Process (AHP). The steps involved in Best Worst method as described by Rezaei (2015) are mentioned below:

Step 1. Determine a set of decision criteria. This can be done through literature review, consulting experts or otherwise. Let the decision-maker identifies *n* criteria {C_1, C_2, C_3, $C_{4,...}C_n$} that are to be used in selecting alternatives.

Step 2. Let the expert determine the best (e.g. most important/vital) and the worst (e.g. least important/trivial) criteria based on the experience.

Step 3. Preference of the best criterion over all the other criteria is determined using a number between 1 and 9. The resulting best-to-others (BO) vector would be: $A_B = (a_{B1}, a_{B2}, ..., a_{Bn})$ where, a_{Bj} shows the preference of the most important i.e. the best criterion B over other criterion j. Hence a_{BB}= 1.

Step 4. Preferences of all criteria over the worst criterion are determined using a number between 1 and 9. The resulting others -to-worst (OW) vector would be: $A_W = (a_{1W}, a_{2W}, ..., a_{nW})^T$ where, a_{jW} shows the preference of the criterion j over the least preferred i.e. worst criterion W. Therefore a_{WW}= 1.

Step 5. Finally the optimal weights $(w_1^*, w_2^*, ..., w_n^*)$ are obtained. Here the objective is to determine the optimal weights of the criteria such that the maximum absolute differences $|\frac{w_B}{w_j} - a_{Bj}|$ and $|\frac{w_j}{w_w} - a_{jW}|$ for all j is minimized, which is translated to the following minmax model:

$$\min \max_j \{ | \frac{w_B}{w_j} - a_{Bj} |, | \frac{w_j}{w_w} - a_{jW} | \}$$

$$s.t.$$

$$\sum_j w_j = 1$$

$$w_j \geq 0, \forall j$$

In BWM it is very easy to check consistency of the comparisons visually using the consistency table and the characteristics of the pair wise vectors of BWM. If $a_{Bj} \times a_{jW}$ is significantly different from the value of a_{BW}, both a_{Bj} and a_{jW} may be revised to reduce the anomaly and make the comparison more consistent. Even if the comparisons are not fully consistent, researchers can select one of the multiple optimal solution obtained (Rezaei, 2015).

2.2 Preference Ranking Optimization Method for Enrichment and Evaluation (PROMETHEE)

PROMETHEE, introduced by Brans (1982) and developed by Brans and Vincke (1985) is an interactive method widely used in multicriteria decision making (MCDM) applications in different fields of study. The input to PROMETHEE is the set of the criteria, their weights, details of the actions (alternatives) with respect to the criteria selected and the preference function. The weights of the criteria can be obtained through various methods however PROMETHEE does not provide any specific recommendations on the process of generating weights (Macharis et al., 2004). A preference function transforms the difference between scores obtained by two alternatives with respect to a particular criterion into a number between 0 and 1. For two particular actions (a) and (b), the preference function of (a) with respect to (b) can be seen as

$$P(a,b) = \begin{cases} 0 \text{ , if } f(a) \leq f(b) \\ \mathrm{P}\left[f(a) \text{ , } f(b) \right], \text{if } f(a) > f(b) \end{cases}$$

Different functions such as usual criterion, level criterion, U-criterion, V-criterion, criterion with linear preference and indifference area, and Gaussian criterion are used (Brans and Vincke, 1985). PROMETHEE is done through two steps:

First alternatives (actions) are partially ranked in PROMETHEE-I using

$$\pi(a, b) = \left(\frac{1}{k}\right) + \sum_{h=1}^{k} P_h\,(a, b)$$

with net outgoing flow as $\Phi^{+}(\mathrm{a})$ where $\Phi^{+}(\mathrm{a}) = \sum_{\mathrm{x}\epsilon\mathrm{k}} \pi(\mathrm{a}, \mathrm{x})$ and net incoming flow as $\Phi^{-}(\mathrm{a})$ where $\Phi^{-}(\mathrm{a}) = \sum_{\mathrm{x}\epsilon\mathrm{k}} \pi(\mathrm{x}, \mathrm{a})$. Total ranking of the alternatives from best to worst are provided in PROMETHEE-II using $\Phi(\mathrm{a})$ where $\Phi(\mathrm{a}) = \Phi^{+}(\mathrm{a}) - \Phi^{-}(\mathrm{a})$. A sensitivity analysis can also be performed to analyze the changes in ranking by the changes in the weights of the criteria.

3. A comparative analysis of BWM and PROMETHEE

Every method has its strengths and weaknesses and both BWM and PROMETHEE are no different. This section presents a comparative analysis of both the methods on underlying principle, ability to structure, consistency assessments, evaluation elicitation and support for group decisions.

3.1 Underlying Principle

BWM can be considered as an aggression method of additive type. Such aggregations generally neutralize good scores of some criteria with bad scores on some other. Important information sometimes gets lost in such aggregations. PROMETHEE-I does a partial ranking of the alternatives in which such a tradeoff is avoided. However PROMETHEE- II, because of aggregation for complete ranking, leads to loss of data.

3.2 Ability to structure

BWM has a clear advantage on ability to structure the problem in a detail hierarchy after disintegrating the problem into smaller sub problems. This helps the decision maker focus better on individual criteria and sub criteria there in. PROMETHEE does not provide such a structuring possibility and hence generating weights for higher number of criteria becomes difficult.

3.3 Consistency assessment

In BWM it is easy to check consistency of the comparisons using the consistency table and the characteristics of the pair wise vectors of BWM. Even if the comparisons are not fully

consistent, researchers can select one of the multiple optimal solution obtained (Rezaei, 2015). Since PROMETHEE does not discuss anything on the process of criteria weight generation, there is no mention on assessment of consistency on the input. However PROMETHEE permits sensitivity analysis and thus checks the highest allowable deviations in criteria weights post facto.

3.4 Evaluation elicitation

BWM uses pairwise comparisons in generating weights for the criteria and in the evaluation of alternatives. The numbers of such comparisons are comparatively higher than PROMETHEE as the evaluations are limited to each alternative on each criterion.

In addition to the number of comparisons, BWM also suffers from the restrictions of a 9 point comparison scale. If a criterion is, say 15 times more important than the other criteria, we do not have a place in the scale. PROMETHEE does not suffer from such an issue. Moreover if an additional alternative has to be added to the existing problem, one needs to put the corresponding values in the PROMETHEE system. For BWM it is a lengthy process as we need to compare this alternative along with others for every criterion available in the problem.

3.5 Support for group decision

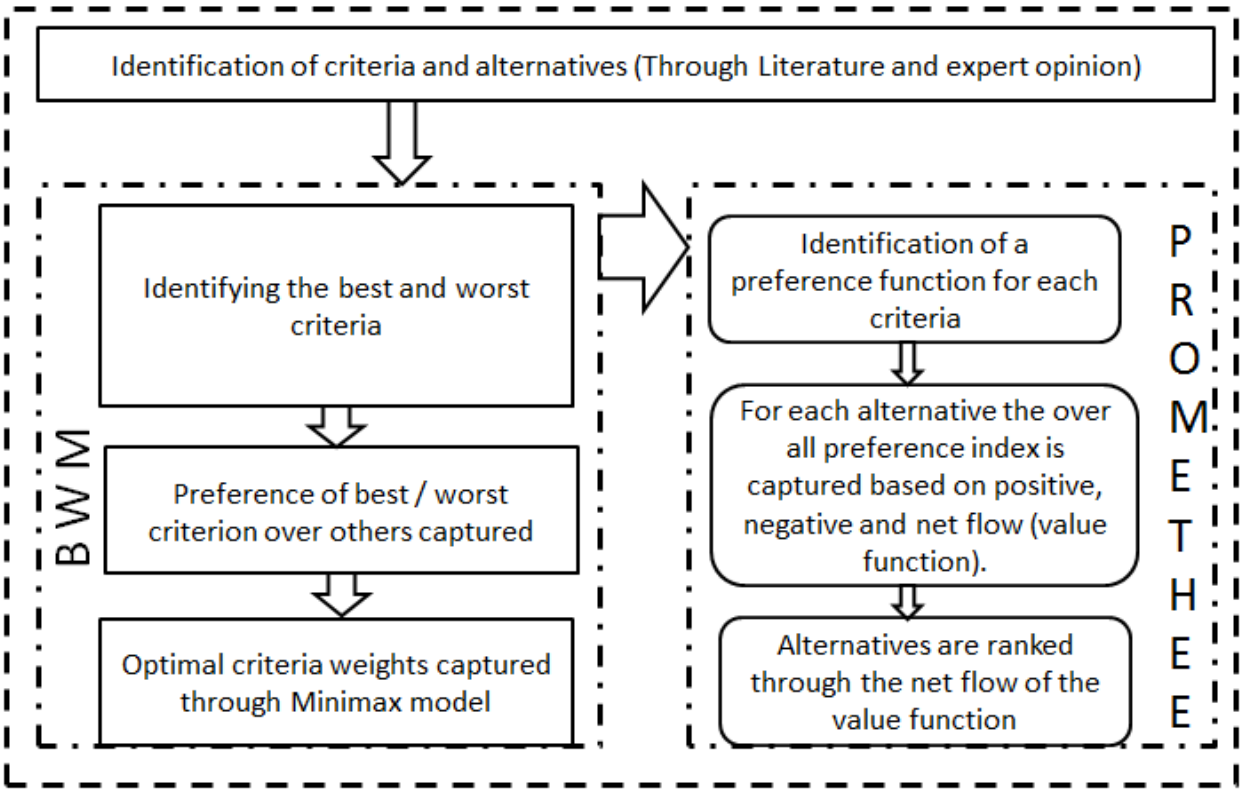
Both BWM and PROMETHEE provide support to group decision making through aggregation. In BWM, arithmetic mean of individual responses is taken whereas in PROMETHEE weighted sum of individual net flows are taken into consideration.

4. Enhancement of PROMETHEE through the characteristics of BWM and proposing BWM-PROMETHEE integrated framework

The ability to decompose the decision problem helps the researcher gain a better overview of the problem. Comparing the best and the worst criterion with respect to the other criteria using a total of (2n – 3) pairwise comparisons (for n criteria), one can generate the weights for the criteria and sub - criteria in BWM which can be integrated to PROMETHEE. This would provide a formal structure to the weight generation process within PROMETHEE with clarity and less amount of computations.

Considering the discussions in section 3 and section 4 we propose the novel BWM – PROMETHEE integrated framework for MCDA. Figure 1 presents the framework.

Figure 1: BWM – PROMETHEE framework in MCDA



5. Comparative analysis between BWM-PROMETHEE and AHP-PROMETHEE

Hens et al (1992) followed by Macharis et al (2004) proposed AHP-PROMETHEE integration in improving the MCDA tool. In their papers they have integrated characteristics of AHP into PROMETHEE in achieving operational synergy. Macharis et al in their paper demonstrated how this integration can benefit multicriteria decision analysis with the hierarchical structure and structured weight generation method. However, the researchers did not provide evidence of it through any real life application. Recently, Best Worst Method (BWM) introduced by Rezaei (2015) demonstrated potentials in reducing volume of calculation compared to Analytic Hierarchy Process (AHP). Interestingly, amalgamation of BWM with PROMETHEE would strengthen PROMETHEE further. AHP had a disadvantage with a very high number of pairwise comparisons in the order of n^2, specifically $\frac{n(n-1)}{2}$. With BWM, the number reduces to (2n – 3) giving it an advantage. Furthermore BWM has an advantage over

AHP as it is more consistent (Rezaei, 2015). Measuring the consistency and rectifying inconsistencies are also easier in BWM and thus giving BWM-PROMETHEE an edge over its rival.

6. A real world application of BWM-PROMETHEE integrated method

In this section we discuss a real world application of the proposed integration. For this purpose we consider a multi criteria decision making problem: Selection of a polytechnic college. Seven polytechnic colleges are evaluated using this BWM-PROMETHEE integration to illustrate the applicability of the proposed method.

6.1 Collection of data

Fifteen experts are consulted and are requested to provide their response in evaluating polytechnic colleges. These experts are the faculty members of polytechnic institutions, hailing from Durgapur and nearby places with an in-depth knowledge of the institutions selected for the study. The average experience of these experts is 10 years with maximum and minimum being 17 years and 5 years respectively. All of them have worked in at least one of the select institutions for a min period 3 years. First, experts are requested to undergo pairwise comparison between criteria and then between sub-criteria within each criteria. Two sets of questionnaires are provided to the experts; one on AHP framework and the other on BWM framework. These pairwise comparisons are done among 'best to other criteria' and 'worst to other criteria' as mentioned in section 2.1. Further these responses are aggregated to generate weights for criteria and sub-criteria. Data for the alternatives are collected from these experts and are put into PROMETHEE framework.

6.2 Selection of criteria and sub criteria for the evaluation of polytechnic Institutions

Five criteria and 15 sub criteria are identified through literature (Chatterjee and Mukherjee, 2010; Bhatt et al., 2013; Chatterjee and Mukherjee, 2013; Mahendran et al., 2014; Wongsathan et al., 2014) and opinion from fifteen experts and are presented in Table 1.

Table 1: Criteria and sub criteria

Criteria >>	**Campus Infra**	**Faculty**	**Student**	**Academic Infra**	**Processes**
Sub criteria	Hostel facility	Qualification &Experience	Admission	Laboratory	Curriculum management
	Sports facility	Faculty retention	Gender diversity	Classroom facility	Internal processes
	Security	Faculty to student ratio	Placement	Library	Evaluation

6.3 Alternatives

The alternative institutions are presented in Table 2. All these polytechnic colleges are privately funded, located in and around the city of Durgapur - the second largest educational hub of West Bengal, and are functioning at least for the last three years.

Table 2: Institutions with abbreviations

Rajendranath College of Polytechnic (RCP)	Bengal college of Polytechnic (BCP)
Dr. B.C. Roy Polytechnic (BCRP)	New Horizons Institute of Technology (NHIT)
Swami Vivekananda School of Diploma (SVSD)	Durgapur Polytechnic College (DPC)
Durgapur Institute of Polytechnic (DIP)	

6.4 Results and Findings

We applied BWM in generating weights of the criteria and sub criteria considered in evaluating the polytechnic colleges. Among criteria, 'Faculty' is found to have highest weight followed by 'Student' and 'Academic infra'. 'Student admission' and 'faculty qualification and experience' emerged as two most important sub criteria. Table 3 presents the weights of all the criteria and sub criteria.

After the weights of the sub-criteria are captured through BWM, they are inducted into PROMETHEE to evaluate and rank the colleges. Table 4 presents complete rankings of the alternatives with respect to net dominance flow 'Phi'. The Phi values corresponding to NHIT and BCRP indicate that these are the only two institutions with positive net flow. DIP though in negative Phi value, is the third institution to look at from the PROMETHEE evaluation. Figure 2 demonstrates the relative position of the alternatives with respect to the net flow and the weights corresponding to the sub-criteria.

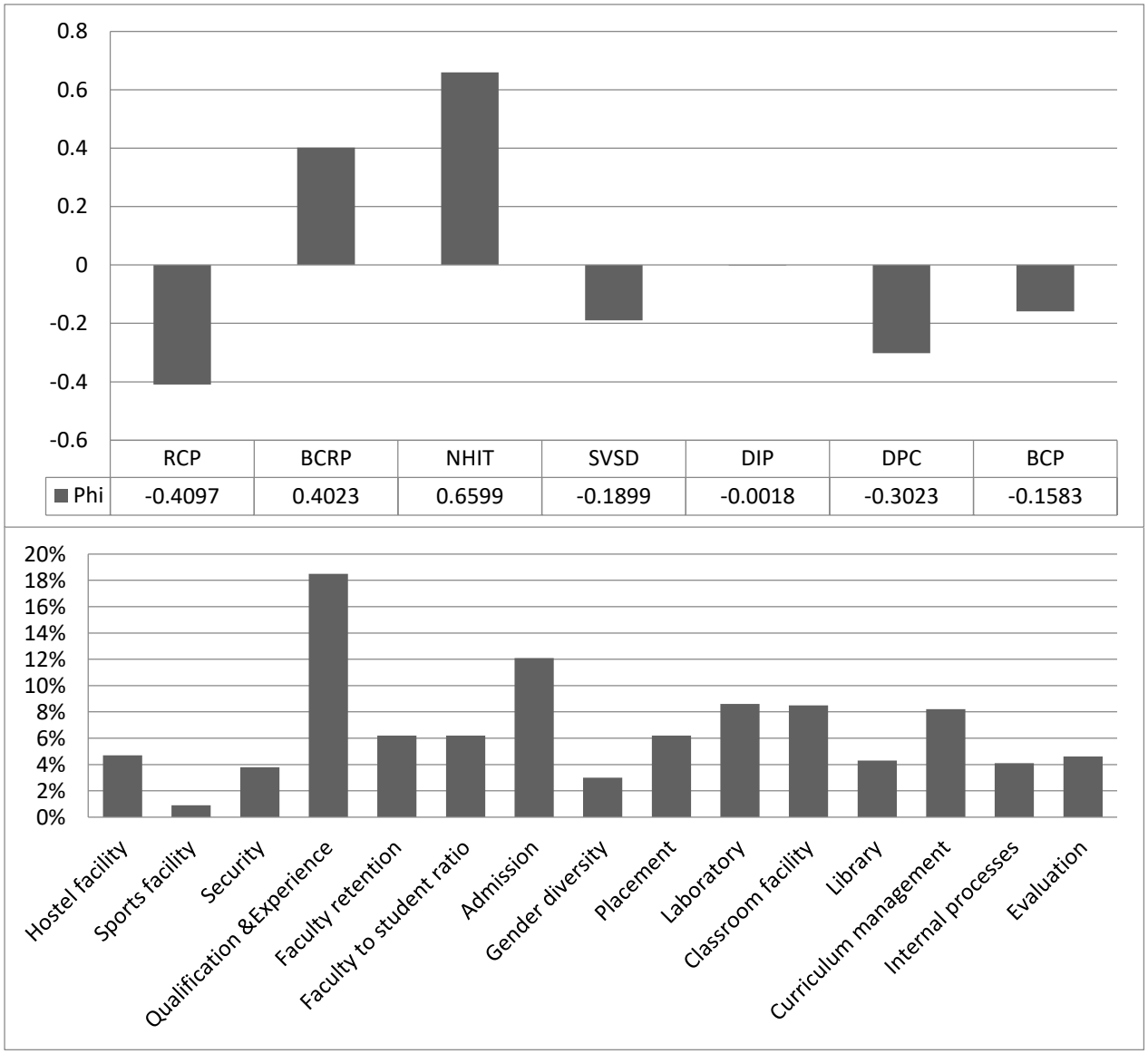
Table 3: Criteria and sub criteria weights

Campus Infra	Hostel facility	0.047
0.0936	sports facility	0.009
	Security	0.038
Faculty	Qualification &Experience	0.185
0.3089	Faculty retention	0.062
	faculty to student ratio	0.062
Student	Admission	0.121
0.2137	gender diversity	0.030
	placement	0.062
Academic Infra	Laboratory	0.086
0.2137	Classroom facility	0.085
	Library	0.043
Processes	Curriculum management	0.082
0.1701	Internal processes	0.041
	Evaluation	0.046

Table 4: Net flow in PROMETHEE

	Phi+	Phi-	Phi
RCP	0.0016	0.4113	-0.4097
BCRP	0.4435	0.0412	0.4023
NHIT	0.6788	0.0189	0.6599
SVSD	0.1010	0.2909	-0.1899
DIP	0.1574	0.1592	-0.0018
DPC	0.0498	0.3521	-0.3023
BCP	0.1085	0.2668	-0.1583

Figure 2: Net flow for the alternatives and criteria weights



Once the BWM-PROMETHEE results are obtained, it is compared with that of AHP-PROMETHEE. The results show a very high correlation, to the extent of 0.938 between the weights generated by AHP and BWM. Figure 3 presents these criteria weight comparison.

Figure 3: Criteria weights comparison between BWM and AHP

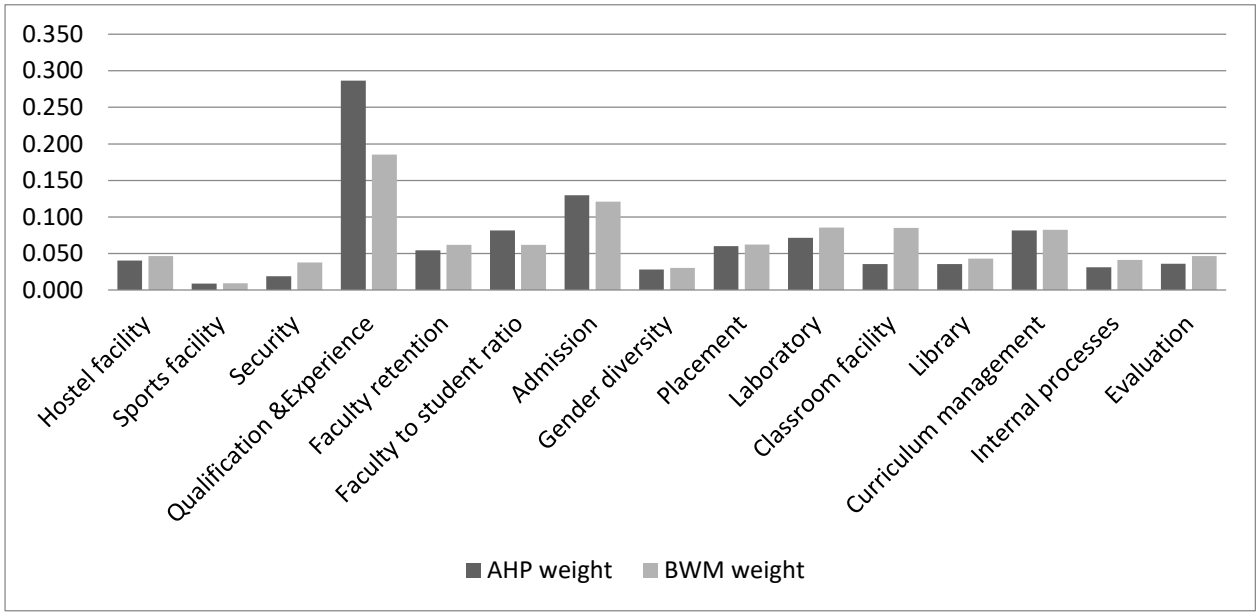
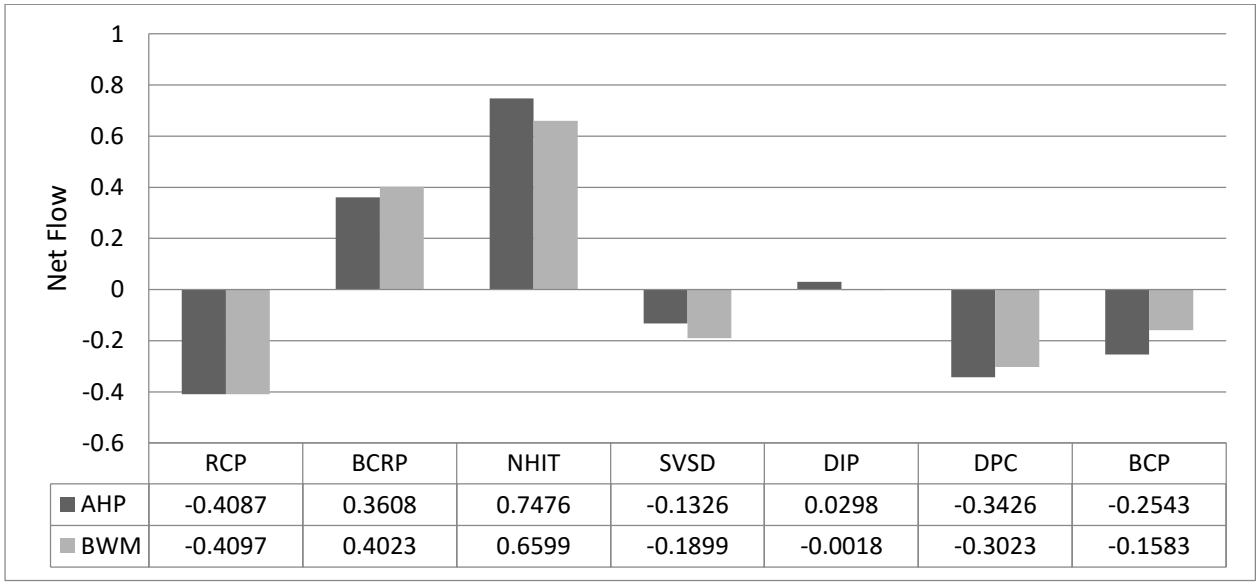


Figure 4 presents a comparison between BWM-PROMETHEE and AHP-PROMETHEE ranks. The result demonstrates no change in the relative positions of these colleges across both methods.

Figure 4: Comparison between BWM-PROMETHEE and AHP-PROMETHEE ranks



	RCP	BCRP	NHIT	SVSD	DIP	DPC	BCP
AHP	-0.4087	0.3608	0.7476	-0.1326	0.0298	-0.3426	-0.2543
BWM	-0.4097	0.4023	0.6599	-0.1899	-0.0018	-0.3023	-0.1583

However there are improvements noticed in BWM-PROMETHEE approach over AHP-PROMETHEE. Firstly, there is 12% reduction in the number of pairwise comparisons recorded in BWM-PROMETHEE approach over AHP-PROMETHEE. Furthermore, this percentage can

increase with the increase in number of criteria. Secondly, in terms of consistency, BWM-PROMETHEE approach has also demonstrated an advantage over AHP-PROMETHEE. Incidentally this improvement on consistency measurements is in line with the claim made by Rezaei (2015) in his paper.

7. Conclusion, limitation and future scope of research

This paper proposes a novel BWM-PROMETHEE integration to strengthen multicriteria decision analysis. This is achieved by integrating couple of important characteristics of BWM into PROMETHEE. The first and the foremost development is the ability to generate weights for the criteria using a formal method and integrate it with PROMETHEE. Moreover, with the ability to reduce the number of pair wise comparisons without compromising the consistency, BWM-PROMETHEE integration thus provides a better deal over AHP-PROMETHEE (Hens et al., 1992; Rezaei, 2015). The second development is in the form of consistency in generating weights. Since BWM weights are always consistent, it has an edge over AHP and thus becomes a better tool when embedded to PROMETHEE. The third development is the ability to overcome the constraint of 9 point scale (Saaty, 2008) and avoid fractional numbers while comparing criteria.

To demonstrate the applicability of the proposed method a real life decision problem – Selection of polytechnic college using data and expert opinion is considered. We also used AHP-PROMETHEE method and compared the results to check deviations, if any. Incidentally we found both the integrated methods, BWM-PROMETHEE and AHP-PROMETHEE provided exactly the same ranking among the colleges, however with less number of pairwise comparisons in BWM-PROMETHEE method.

This paper attempted to combine strengths of two multicriteria decision making methods in overcoming their individual weaknesses. Researchers and policy makers can use this integrated multicriteria decision making method across other disciplines to overcome challenges with voluminous computations. Though the excel solver could handle the volume of data for this piece of work, an efficient software would help future researchers in handling larger data with ease and efficiency.

References

Athawale. V.M.. Chatterjee. P. and Chakraborty. S. (2012) Decision making for facility location selection using PROMETHEE II method. International Journal of Industrial and Systems Engineering. 11 (1/2). pp. 16 - 30

Bhatt, R., Patel, B., & Bhatt, D. (2013). Best Engineering College Selection Through Analytic Hierarchy Process: A Case Study. International Journal of Technology and Engineering Education, 23.

Brans. J.P. and Vincke. Ph. (1985) A preference ranking organization method. The PROMETHEE method for MCDM. Management Science. 31(6). pp. 647-656.

Chatterjee, D., & Mukherjee, B. (2010). Study of fuzzy-AHP model to search the criterion in the evaluation of the best technical institutions: a case study. International Journal of Engineering Science and Technology, 2(7), 2499-2510.

Chatterjee, D., & Mukherjee, B. (2013). A study of the comparison of AHP and Fuzzy AHP evaluations of private technical institutions in India. IJITR, 1(4), 283-291.

Chitsaz, N., & Azarnivand, A. (2016). Water Scarcity Management in Arid Regions Based on an Extended Multiple Criteria Technique. *Water Resources Management*, 1-18.

Frikha. H.. Chabchoub. H. and Martel. J-M. (2011) An interactive disaggregation approach inferring the indifference and the preference thresholds of PROMETHEE II. International Journal of Multicriteria Decision Making. 1 (4). pp. 365 - 393

Ghaffari, S., Arab, A., Nafari, J., & Manteghi, M. (2017). Investigation and evaluation of key success factors in technological innovation development based on BWM. *Decision Science Letters*, *6*(3), 295-306.

Guo, S., & Zhao, H. (2017). Fuzzy best-worst multi-criteria decision-making method and its applications. *Knowledge-Based Systems*.

Linkov, I., Varghese, A., Jamil, S., Seager, T. P., Kiker, G., & Bridges, T. (2004). Multi-criteria decision analysis: a framework for structuring remedial decisions at contaminated sites. In *Comparative risk assessment and environmental decision making* (pp. 15-54). Springer Netherlands.

Mahendran, P., Moorthy, M. B. K., & Saravanan, S. (2014). A fuzzy AHP approach for selection of measuring instrument for engineering college selection. Appl. Math. Sci, 8(44), 2149-2161.

Mou, Q., Xu, Z., & Liao, H. (2016). An intuitionistic fuzzy multiplicative best-worst method for multi-criteria group decision making. *Information Sciences*, *374*, 224-239.

Ren, J., Liang, H., & Chan, F. T. (2016). Urban sewage sludge, sustainability, and transition for Eco-City: Multi-criteria sustainability assessment of technologies based on best-worst method. Technological Forecasting and Social Change.

Rezaei, J., Nispeling, T., Sarkis, J., & Tavasszy, L. (2016). A supplier selection life cycle approach integrating traditional and environmental criteria using the best worst method. *Journal of Cleaner Production*, *135*, 577-588.

Rezaei J. (2015) Best-worst multi-criteria decision-making method. Omega. 53. pp. 49-57

Rao, R. V. (2007). *Decision making in the manufacturing environment: using graph theory and fuzzy multiple attribute decision making methods*. Springer Science & Business Media.

Saaty, T. L. (2008). Decision making with the analytic hierarchy process. *International journal of services sciences*, *1*(1), 83-98.

Walther. G.. Spengler. T. and Queiruga. D. (2008) Facility location planning for treatment of large household appliances in Spain. International Journal of Environmental Technology and Management. 8(4). pp. 405 - 425.

Wongsathan, R., Khaothawirat, A., & Khuankaew, W. (2014, July). The prioritization for higher education institutions performance criteria with Fuzzy Analytical Hierarchy Process. In 2014 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE) (pp. 1395-1401). IEEE.